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Electron-electron scattering and conductance of long many-mode channels

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ABSTRACT

The electron-electron scattering increases the resistance of ballistic many-mode channels whose width is smaller than their length. We show that this increase saturates in the limit of infinitely long channels. Because the mechanisms of angular relaxation of electrons in three and two dimensions are different, the saturation value of the correction to the resistance is temperature-independent in the case of three-dimensional channels and is proportional to the temperature for two-dimensional ones. The spatial behavior of electron distribution in the latter case is described by an unusual characteristic length.

1. Introduction

Though the electron-electron scattering does not directly contribute to the electrical resistance in the absence of umklapp processes [1], it affects the current in small-size conductors. In particular, it leads to a minimum in the temperature dependence of the resistance [2] of a wire with diffusive boundary scattering due to the electronic analogues of Knudsen [3] and Poiseuille effects. The latter represents a decrease of resistance with increasing temperature due to decreasing viscosity of the electron liquid and is also known as the Gurzhi effect [4]. A similar decrease of resistance was obtained later for 2D constrictions with viscous electron flow [5], where the electron-electron scattering serves as a ”lubricant” for the rough boundaries of the conducting area. The electron-electron scattering results in the decrease of the resistance even for contacts with smooth boundaries because it changes the trajectories of electrons and may prevent them from passing through the constriction or help them to get through it [6,7]. This decrease was experimentally observed in several papers [8,9].

As the electron-electron collisions conserve the total momentum of electrons, they may affect the conductance only in the presence of a spatial inhomogeneity that absorbs or provides the extra momentum. In the above cases, this inhomogeneity was represented by the hard boundaries of the conducting area, but the extra momentum may be also absorbed by the electron reservoirs at the ends of any conducting system of a finite size. This suggests that the electron-electron scattering may affect the current in finite-length conducting channels even in the case of a specular reflection from the walls. Recently, the correction to the conductance of a narrow multichannel ballistic conductor was calculated for the weak electron-electron scattering [10]. This correction appeared to be negative and resulted from pairwise collisions that changed the number of electrons moving to the right and to the left, i.e. whose projection of the velocity on the channel axis was positive or negative (see Fig. 1). In any dimension higher than 1, these collisions are allowed by the conservation laws. If an electron originating from one of the reservoirs is scattered back into the same reservoir, it does not contribute to the current and hence the resistance of the channel increases [11].

As the calculations in Ref. [10] were performed in the lowest approximation in the electron-electron scattering, the resulting correction to the conductance was proportional to the length of the channel. However, it was not clear whether the conductance tends to zero with increasing length of the channel or stops to decrease at some finite value. The purpose of the present paper is to calculate the correction to the conductance in the limit of strong electron-electron scattering.

The correction to the electric current is determined by the angular relaxation of electron distribution, which is essentially different in three-dimensional (3D) and two-dimensional (2D) electron gases [12,13]. The 3D relaxation is dominated by small-angle scattering and therefore all angular harmonics, both odd and even, decay with the same characteristic time. In contrast to this, the 2D relaxation has a significant contribution from large-angle scattering that results from collisions of electrons with almost opposite momenta, and this results in strongly different relaxation times of the symmetric and antisymmetric parts of the distribution function in the momentum space. The angular relaxation of the symmetric part in the 2D case is determined by the collisions of electrons with almost opposite momenta, which rotate the pair of excess electrons in the momentum space about the origin, and results in the relaxation rate proportional to T^2. In the case of the antisymmetric part, an excess electron on one side of the Fermi surface has no pair on its opposite side, and therefore this mechanism does not work. Instead the relaxation of this part proceeds through small-angle scattering and its rate is proportional to T^4, which is much smaller than

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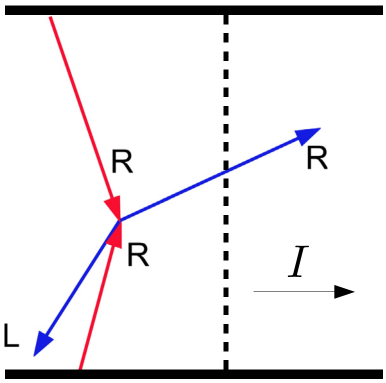

Fig. 1. A collision of two electrons that changes the number of right-movers. One of the right-movers is converted into a left-mover despite the momentum conservation.

T^2 at low temperatures. Because the odd and even angular harmonics of the electron distribution are coupled in a spatially inhomogeneous system, determining the temperature dependence of the correction to the conductance of the 2D channel is an interesting question.

The calculation of the correction to the conductance of a long channel presents a nontrivial mathematical problem that cannot be solved by standard methods of kinetic theory. The first reason is that the calculation of the current involves a large number of angular harmonics of the electron distribution and not only the lower ones as in bulk conductors. The second reason is that the electron distribution exhibits a different behavior in different portions of the channel. While it is almost constant in its middle part, it sharply changes near its ends, and it is difficult to describe its spatial dependence using the same approximations everywhere. To overcome these difficulties, a custom semi-analytical approach is used in this paper.

The paper is organized as follows. In Sec. 2 we present the model and basic equations, in Sec. 3 we perform calculations for the 3D case, and Sec. 4 presents calculations for the 2D case. In Sec. 5 we discuss the results in terms of physics, and Sec. 6 presents the summary. Appendices contain more details of calculations.

2. Model and basic equations

Consider a metallic wire of a uniform cross-section that connects two electronic reservoirs. We assume that the length L of the wire is much larger than its transverse dimensions, and these dimensions are much larger than the Fermi wavelength. There are no impurities in the wire, and the boundaries are assumed to be absolutely smooth so that the electrons are specularly reflected from them and their longitudinal momentum is conserved. The narrowness of the channel allows us to neglect the effects of electron-electron scattering outside the channel because they are proportional to the number of transverse quantum modes squared [6].

The distribution function of electrons in the channel obeys the Boltzmann equation

$$\frac{\partial f}{\partial t} + v\,\frac{\partial f}{\partial r} + eE\,\frac{\partial f}{\partial p} = \hat{I}_{ee}, \tag{1}$$

where $E = -\nabla\phi$ is the electric field and the electron–electron collision integral $\hat{I}_{ee}$ is given by

$$\begin{aligned}\hat{I}_{ee}(p) = \alpha_{ee}\, \nu_d^{-2} \int \frac{d^dk}{(2\pi)^d} \int \frac{d^dp'}{(2\pi)^d} \int d^dk' \\ \times \delta(p+k-p'-k')\,\delta(\varepsilon_p+\varepsilon_k-\varepsilon_{p'}-\varepsilon_{k'}) \\ \times \{[1-f(p)]\,[1-f(k)]\,f(p')\,f(k') \\ -f(p)\,f(k)\,[1-f(p')]\,[1-f(k')]\}\,,\end{aligned} \tag{2}$$

α_{ee} is the dimensionless interaction parameter, $d = 2$ or 3 is the dimensionality of the system; $\nu_3 = mp_F/\pi^2$ and $\nu_2 = m/\pi$ are the three- and two-dimensional two-spin electronic densities of states ($\hbar = 1$). The assumption of momentum-independent interaction parameter is valid if the screening length of the electron-electron interaction is sufficiently short. This can be ensured by a high enough concentration of electrons in the 3D case or by a close electrostatic gate in the 2D case. The current through an arbitrary section of the conductor is given by an integral over the transverse coordinates

$$I = 2e \int d^{d-1}r_\perp \int \frac{d^dp}{(2\pi)^d}\, v_x\, f(p, x, r_\perp). \tag{3}$$

Because of the condition $E_F \gg \max(eV, T)$ one may treat the electron velocity near the Fermi surface as energy independent and set $v = v_F n$, where n is a unit vector in the direction of p. It is possible to avoid solving the Poisson equation for the electric potential ϕ if one replaces p as the argument of f by n and the energy variable $\varepsilon = \varepsilon_p + e\phi(r) - E_F$. With the new variables, the term with electric field drops out from Eq. (1), and it takes up the form

$$\frac{\partial f(n,\varepsilon,r)}{\partial t} + v\,\frac{\partial f}{\partial r} = \hat{I}_{ee}\{f\}|_{n,\varepsilon,r}. \tag{4}$$

The boundary conditions for this equation at the left and right ends of the channel are

$$f(\varepsilon,\ n_x > 0,\ x = 0) = f_0\,(\varepsilon - eV/2)\,, \tag{5}$$

$$f(\varepsilon,\ n_x < 0,\ x = L) = f_0\,(\varepsilon + eV/2)\,, \tag{6}$$

where x is the longitudinal coordinate, V is the voltage drop across the channel, and $f_0(\varepsilon) = 1/[1+\exp(\varepsilon/T)]$ is the equilibrium Fermi distribution function.

Because we are interested in the electric current, the angular relaxation of electrons will be of primary importance to us. As the physics of this relaxation is essentially different in 3D and 2D electron gases, one has to make the different approximations for these cases, and in what follows we treat them separately.

3. 3D channel

In the case of a 3D channel, the angular relaxation is dominated by small-angle scattering $|\Delta p| \ll p_F$, and therefore all angular harmonics have nearly the same relaxation time $\tau^{-1} \sim T^2/E_F$ [12,14]. The exceptions are the spherical harmonics with $l = 0$ and $l = 1$, which have zero relaxation rates because of the particle-number and momentum conservation laws. We assume that the channel is cylindrically symmetric and linearize Eq. (4) with respect to the voltage drop assuming $eV \ll T$ by a substitution [15]

$$f(n,\varepsilon,x) = f_0(\varepsilon) + f_0\,(1-f_0)\,\psi(x,n), \tag{7}$$

where x is the longitudinal coordinate and $\psi(x,n)$ describes the angular distribution of electrons. As the relaxation of all angular harmonics with $l > 1$ may be approximately described by a single characteristic time τ, one may subtract the harmonics with $l < 2$ from ψ in the collision integral and write down Eq. (4) for ψ in the form

$$v_x\,\frac{\partial\psi}{\partial x} = -\frac{1}{\tau}\,(\psi - \overline{\psi} - \psi_1), \tag{8}$$

where $\overline{\psi}$ and ψ_1 are the zero and first harmonics of ψ given by the angular integrals

$$\overline{\psi}(x) = \int \frac{d\Omega}{4\pi}\,\psi(x,\theta), \tag{9}$$

$$\psi_1(x,\theta) = 3\cos\theta \int \frac{d\Omega'}{4\pi}\,\cos\theta'\,\psi(x,\theta'), \tag{10}$$