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Impurity-directed transport within a finite disordered lattice

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ABSTRACT

We consider a finite, disordered 1D quantum lattice with a side-attached impurity. We study theoretically the transport of a single electron from the impurity into the lattice, at zero temperature. The transport is dominated by Anderson localization and, in general, the electron motion has a random character due to the lattice disorder. However, we show that by adjusting the impurity energy the electron can attain quasi-periodic motions, oscillating between the impurity and a small region of the lattice. This region corresponds to the spatial extent of a localized state with an energy matched by that of the impurity. By precisely tuning the impurity energy, the electron can be set to oscillate between the impurity and a region far from the impurity, even distances larger than the Anderson localization length. The electron oscillations result from the interference of hybridized states, which have some resemblance to Pendry's necklace states (Pendry, 1987) [21]. The dependence of the electron motion on the impurity energy gives a potential mechanism for selectively routing an electron towards different regions of a 1D disordered lattice.

1. Introduction

Many researchers [1–9] have studied open (infinite) models of one-dimensional regular lattices, in which an impurity is introduced that allows for control over transport and closely related properties in the lattice. This has led us to consider the possibility that an impurity might be used to control transport even in disordered finite systems, with one question in mind: What type of transport would occur if the ordered lattice were replaced by a disordered one?

It is well-known that disorder in quantum systems produces Anderson localization [10]. There have been numerous theoretical and experimental studies on Anderson localization [11–15]. For example, on the theoretical side, it has been shown that in a one-dimensional lattice with random energies at each site, all the eigenstates of the Hamiltonian are localized [16–18]. Although this result indicates that there can be no electron conductance through an *infinite* one-dimensional disordered lattice, sharp resonances at the band center have been noted [19,20]. Such resonances are required for electron transport. In fact, Pendry [21] has shown that it is possible to transmit an electron from one end of a disordered *finite* lattice to the other due to the presence of "necklace states" that serve as stepping-stones for the electron. Necklace states also exist in optical systems [22,23]. These states form a sub-band that can induce resonant transport similar to the energy band of an ordered lattice. Resonances of finite disordered systems coupled to infinite reservoirs have been theoretically studied in [24,25].

Extrapolating from these previous studies, here we consider finite disordered lattices (or quantum wires) with a side-attached impurity ("T-junction"). The impurity can be realized using a quantum dot, which constitutes a nano-control device. The properties of the dot can be altered through a gate potential allowing an experimentalist control over electron transport. Varying the gate potential on the dot can be used to probe the spectrum and localization properties of the lattice. As we will show, we can indeed use an impurity to direct transport within a disordered lattice. In our theoretical study we will consider the case of zero temperature. Therefore the transport we will discuss is different from variable-range hopping [26,27], which occurs at non-zero temperature. We will discuss possible extension of our work to the case of nonzero temperature in Section 7. Note that our lattice is finite, but large enough so that boundary effects only play a minor role.

Experimentally, effective 1-D systems can be synthesized by a variety of techniques [28,29], including lattice geometries that incorporate a side-attached quantum dot [30,31]. Randomized site potentials in a finite lattice might be obtained, for example, by varying segment lengths (i.e., growth times) in GaAs/GaP superlattices assembled by laser-assisted catalytic growth [15,29]. An effective side-attached dot could then potentially be introduced by doping one such segment.

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While as far as we are aware there have been no studies on disordered models of electron transport incorporating a side-attached impurity, at least one experimental realization of a system similar to ours has been reported in Ref. [32] using microwaves instead of electrons. The system in Ref. [32] consists of a waveguide with random blocks (analogous to our disordered lattice) and an air-gap in the middle (analogous to our impurity site). The focus of Ref. [32] however is different from the focus of our present study, as we will discuss below. In another relevant work, boundary effects on localization properties have recently been studied in finite, weakly disordered optical waveguide arrays [34]. Moreover, Refs. [27,33] have considered control of thermopower using a gate potential that shifts all the lattice energies at once.

We will consider the motion of the electron from the impurity to the lattice and back. The initial state is that in which the electron is completely localized in the side impurity. Hereafter this state will be referred to as the unperturbed *impurity state*; this state is an eigenstate of the unperturbed Hamiltonian, corresponding to the case where the impurity is decoupled from the lattice. The coupling will allow the electron to transfer between the impurity and the lattice.

We will treat the energy of the impurity as a tunable parameter. We will study how this parameter influences the transport of the electron from the impurity to the lattice, or vice versa.

Our main finding is that for certain impurity energies the electron can jump to small regions in the lattice; these regions are localization centers of Anderson-localized states whose energy is matched by the impurity energy. These, together with the impurity state, form hybridized states that are similar to the necklace states studied by Pendry. Interference between the hybridized states induces Rabi-like oscillations of the electron survival probability at the impurity. Hence, the electron alternates positions between the impurity and the localization centers of the lattice states hybridized with the impurity state.

The Rabi-like oscillations occur in the vicinity of avoided crossings in the energy spectrum of the system; these avoided crossings are induced by the interaction between the impurity and the lattice. The center of the avoided crossings signals the appearance of maximally hybridized states. Experimental observation of avoided crossings and hybridized states similar to ours was the main focus of Ref. [32] mentioned above. The new aspect of our study relative to Ref. [32] is the description of the time evolution of the electron associated with these avoided crossings and the possibility of tuning the impurity energy to predictably route electrons to different regions of the lattice. In addition, we will also point out that the range of electron transport can be larger than the localization length of the hybridized states, as long as the impurity energy is precisely tuned to match the center of the avoided crossing.

We will focus our attention on Rabi-like oscillations involving only two or three hybridized states. Oscillations involving many hybridized states produce an erratic pattern of motion, which is less suitable for controlled transport.

The paper is organized as follows: in Section 2 we introduce our T-junction model and discuss the general eigenstate characteristics both before and after the disordered lattice is coupled to the side impurity. In Section 3 and Appendix A we analyze the hybridization properties of the T-junction model in closer detail, before introducing the essential concepts for our transport scheme in Section 4. In Section 5 we discuss how the range of transport can be tuned even beyond the Anderson localization length of hybridized states close to the impurity. Then in Section 6 we show our results are robust against ensemble averaging before presenting our final discussion in Section 7.

2. T-Junction lattice

We consider a T-junction lattice, consisting of a disordered lattice (a finite one-dimensional chain of quantum-wells with random energy levels) and a side impurity attached to one of the wells. The impurity is introduced as a nano-control device that will enable directed electron transport between the impurity and a lattice segment.

We will focus on the motion of a single electron and will neglect Coulomb interactions altogether. We will model the lattice using a tight-binding Hamiltonian with uniform nearest-neighbor interactions, represented as a sum of lattice and impurity Hamiltonians, $H = H_{\text{lattice}} + H_{\text{d}}$. The lattice Hamiltonian is written as

$$H_{\text{lattice}} = \sum_{x=1}^{N} \epsilon_x |x\rangle\langle x| - \frac{b}{2} \sum_{x=1}^{N-1} (|x+1\rangle\langle x| + |x\rangle\langle x+1|) \tag{1}$$

The energies ϵ_x are random energies uniformly distributed to introduce purely diagonal disorder. They describe unoccupied levels of the quantum wells that will roughly form an energy band. The width of the disorder W is represented by the range $W = \epsilon_{\max} - \epsilon_{\min}$, where for simplicity we will set $\epsilon_{\max} = W$ and $\epsilon_{\min} = 0$. Other parameters include the number of lattice sites N and nearest neighbor interaction strength $b/2$. We will choose $W = b$ such that the disorder width is comparable to the nearest-neighbor interaction strength.

The impurity Hamiltonian is given by

$$H_{\text{d}} = \epsilon_d |d\rangle\langle d| - g(|a\rangle\langle d| + |d\rangle\langle a|) \tag{2}$$

The impurity is denoted as d while the lattice attachment site is defined as site a, where $a \in \{1, N\}$; ϵ_d represents the energy of the impurity, which we treat as a tunable parameter. The impurity could be physically realized by using a quantum dot with a variable gate potential [30,31] or by segment doping, although the impurity energy would be fixed for an individual lattice in the latter case. Tunneling strength between the impurity and the attachment site is given by g. We limit our numerical analysis to lattices of size $N = 100$ (Fig. 1).

2.1. Characteristics of uncoupled disordered lattice

To better understand the capability of the side impurity to direct transport within the lattice we first investigate the influence on the spectrum of the Hamiltonian as we vary the tunneling strength. We will begin by investigating the $g = 0$ case, when the lattice and impurity are uncoupled. For this case the Hamiltonian of the disconnected lattice can be diagonalized as

$$H_{\text{lattice}} = \sum_{m=1}^{N} E_m |\psi_m\rangle\langle\psi_m|. \tag{3}$$

The presence of disorder results in Anderson Localization (AL) in the lattice. To demonstrate the occurrence of state localization we numerically diagonalized a specific realization of the lattice Hamiltonian with random site-energies. Fig. 2 shows one of the resulting localized states. In this section and in Sections 3–5 we will use this specific realization of the site energies to illustrate our results.

The degree of state localization can be determined by the second moment of probability density, the inverse participation number [35]

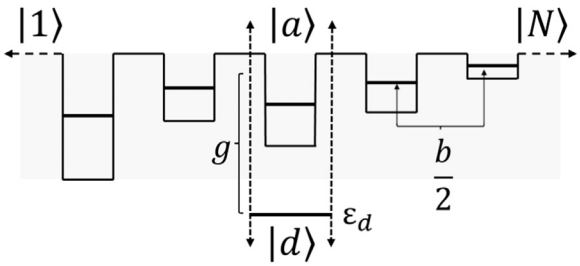

Fig. 1. Zoomed in view of our T-junction lattice in the vicinity of the impurity site d. The total lattice has N sites. The lattice sites have disordered energies within the range W and a constant nearest neighbor interaction energy of $b/2$. The impurity has energy ϵ_d and is attached to the lattice at site a through tunneling strength g.