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Rotation operator approach for the dynamics of non-dissipative multi-state Landau–Zener problems: Exact solutions

M.E. Ateuafack [a], J.T. Diffo [a,b,*], L.C. Fai [a], M.N. Jipdi [a]

[a] Mesoscopic and Multilayer Structures Laboratory, Department of Physics, Faculty of Science, University of Dschang, Cameroon
[b] Department of Physics, Higher Teachers' Training College, The University of Maroua, PO BOX 55 Maroua, Cameroon

HIGHLIGHTS

- Exact solutions for the dynamics of the noisy LZ model are derived.
- The solutions are obtained regardless the initial configuration of the system.
- Stokes constants are expressed through the parabolic cylinder function.
- Euler angles are derived if the 3D-noise correlation function is given.
- Use states of an atom to store information reduce the decoherence effect.

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ABSTRACT

The paper investigates exact time-dependent analytical solutions of the Landau–Zener (LZ) transitions for spin one-half subjected to classical noise field using rotation operator approach introduced by Zhou and co-authors. The particular case of the LZ model subjected to colored noise field is studied and extended to arbitrary spin magnitude. Transition probabilities are derived regardless of the initial configuration of the system and are found to be functions of the sort for Stokes constant. It is observed that the latter may be completely evaluated provided we have knowledge of the phase difference between noise in $x-$ and $y-$directions. Transition probabilities are found to depend not only on the LZ parameter and noise frequency, but also on the states involved in the study. In particular, the coherence of the system is sustained for an exceedingly long time when many levels are considered in an atom and if in addition, the LZ parameter tends to unity and the noise' frequency is low.

1. Introduction

The ability to manipulate and control spins in noisy environment has become a fundamentally challenging topic with a potentially high impact for future realization of quantum devices. This fascinating field is at the origin of new topics developed recently in some physics domains such as spintronics and molecular electronics [1–3]. These areas of modern physics aim to understand and explain the mechanisms of spin transition dynamics in static and dynamic coupling systems and transportation in quantum optics and relevant multi-level systems. For instance, Landau Zener (LZ) transitions [4–7], spin-flip in nanomagnets [8], Bose–Einstein condensates in optical lattices [9] and adiabatic computing [10–20] where the environment is considered as a source of classical noise [21–23], quantum information technology [24,25] recently received great attention. In quantum information processing, a spin $-\frac{1}{2}$ particle is a suitable two-level system. It is the basic unit of binary quantum information [26–30] commonly known as the quantum bit or more simply the qubit.

Numerous realizations of two- and multi-level LZ transitions were observed in recent quantum transport experiments [31–36]. Accordingly, large amount of literature have been devoted to the dynamic of a two-level system driven around an avoided crossing, particularly in connection with LZ physics. Asymptotical [4–7,21,22,16,37–39] as well as time-dependent [23] LZ transition probabilities and related physical quantities were derived in different contexts using several methods. However, most results concerning the time-dependent LZ problem are obtained numerically. Few analytical expressions for the time-dependent standard LZ model [22] subjected to low-frequency colored noise

* Corresponding author at: Department of Physics, Higher Teachers' Training College, The University of Maroua, PO BOX 55 Maroua, Cameroon.
E-mail addresses: jauresdiffo@gmail.com, diffojaures@yahoo.com (J.T. Diffo).

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limit are now available [23]. It is instructive to note that all these results were obtained via some initial conditions imposed to the quantum system. Experimentally, this is not an easy task because accurate control of spin is still a challenge for both theoreticians and engineers. Here, we found a way out of solving the problem without preparing the system from some initial conditions.

This paper is another contribution to the analysis and understanding of the time-dependent non dissipative LZ problem that is generalized for the multi-level problem. With the help of the rotation operator approach detailed in Ref. [40] we derive for the first time the exact time-dependent transition probabilities for the two and multi-level LZ model in random noise field. This is independent of the noise regime and the initial configuration of the quantum system. Our discussion is reported in terms of relaxation and dephasing. The motivation behind the mathematical developments implemented in this paper resides also on the derivation of the Stokes constant and phase [41] which help to tackle inelastic collision and give consistent results for non-adiabatic probabilities.

The present paper is organized as follows: in Section 2, considering the LZ model subjected to arbitrary transverse noise field, we derive and solve the master equation governing the dynamics of Eulers' angles. The resolution of this equation in the special case of colored noise is considered in Section 3. We also investigate the transition dynamics of spin $-\frac{1}{2}$ system. Section 4 presents the case where an arbitrary spin is subjected to a classical transverse noise with Gaussian realizations. Section 5 is the conclusion.

2. Model Hamiltonian and general equation

The model describes a quantum system consisting of two states where level crossings are induced by a classical noise having both diagonal and off-diagonal fluctuating matrix elements. The total Hamiltonian of the system reads:

$$\hat{H} = 2\xi t\sigma_z + \sum_{\ell} g_{\ell}(t)\sigma_{\ell}, \quad \ell = (x, y). \tag{1}$$

Here σ_ℓ are Pauli matrices, $\xi > 0$ is the constant sweep velocity and $g_\ell(t)$ the noise fields.

The relevance of this model comes from the fact that its Hamiltonian (1) contains only terms linear in the spin operator components and thus, obeys a SU(2) algebra. This implies that the system may be treated as some rotations in a three-dimensional space; hence, the propagator of the system can be written in a form described by the Wigner rotation operator in which all the dynamic information is included [42–45]:

$$\hat{U}(t, t_0) \equiv \hat{U}(\alpha, \beta, \gamma) = \exp\left(i\frac{\gamma(t)}{2}\sigma_z\right)\exp\left(i\frac{\beta(t)}{2}\sigma_y\right)\exp\left(i\frac{\alpha(t)}{2}\sigma_x\right) \tag{2}$$

where $\alpha(t)$, $\beta(t)$, $\gamma(t)$ are the three time-dependent Euler angles.

Applying the rotation operator approach, Zhou and coauthors [40,45] cast this problem into three differential equations:

$$\begin{cases} \dot{\gamma}(t) + \dot{\alpha}(t)\cos\beta(t) + g_z(t) = 0 \\ \dot{\beta}(t) + g_x(t)\sin\gamma(t) + g_y(t)\cos\gamma(t) = 0 \\ \dot{\alpha}(t)\sin\beta(t) - g_x(t)\cos\gamma(t) + g_y(t)\sin\gamma(t) = 0. \end{cases} \tag{3}$$

The most important feature of this approach is that it leads to a straightforward calculation of accumulated phases, probabilities of spin transitions and coherence evolutions of the system. Following suitable recombinations and transformations, the Euler kinematic equations (3) can be recast into the Riccati equation [46]:

$$\dot{f} + \frac{i}{2}g_+(t)f^2 + ig_z(t)f - \frac{i}{2}g_-(t) = 0 \tag{4}$$

where the dot stands for time derivative and $g_\pm(t) = g_x(t) \pm ig_y(t)$, $g_z(t) = 2\xi t$ and

$$f(t) = -\tan\left(\frac{\beta(t)}{2}\right)\exp(i\gamma(t)). \tag{5}$$

We introduce the transformations

$$f(t) = \tilde{f}(t)\exp\left\{-i\int_{t_0}^{t} g_z(t')dt'\right\} \tag{6}$$

and

$$\Omega_\pm(t) = g_\pm(t)\exp\left\{\pm i\int_{t_0}^{t} g_z(t')dt'\right\} \tag{7}$$

and write the differential equation (4) in the form

$$\dot{\tilde{f}} + \frac{i}{2}\Omega_+(t)\tilde{f}^2 - \frac{i}{2}\Omega_-(t) = 0. \tag{8}$$

Letting

$$\tilde{f}(t) = \frac{2i}{Z(t)\Omega_+(t)}\frac{dZ}{dt}, \tag{9}$$

then

$$\frac{dZ}{dt} = -\frac{1}{4}\int_{t_0}^{t} g_+(t)g_-(t')\exp\left\{-i\xi\left(t^2 - t'^2\right)\right\}Z(t')dt' \tag{10}$$

which can be interpreted as the general equation governing the time variation of Euler's angles for a two-level system subjected to a classical random field noise. For a well-defined noise correlation function, Eq. (10) is computed and the three Euler angles are obtained provided the complex function $f(t)$ derived from the aforementioned relationships.

3. Dynamics of the Euler angles for a two-level system subjected to a classical noise with Gaussian realizations

In this section, we suppose the system described by the Hamiltonian (1) subjected to a classical noise that obeys the Markovian Gaussian process specified by the time correlation functions

$$\left\langle g_\ell(t)g_{\ell'}(t')\right\rangle = J^2\delta_{\ell\ell'}\exp(-\Upsilon|t - t'|). \tag{11}$$

Here $\langle\cdots\rangle$ denotes the statistical average, J is the averaged amplitude of the diagonal and off-diagonal coupling and Υ is the decay constant.

An alternative way to solve the master equation derived above consists on averaging Eq. (10) over all possible realizations of the considered system. Hence we show that

$$Z(t) = A_-\exp\left\{-\frac{i}{4}\left(\xi t^2 - 2i\Upsilon t - \frac{\Upsilon^2}{4\xi}\right)\right\} \times D_{-i\lambda}\left(t\sqrt{\xi} - i\frac{\Upsilon}{\sqrt{\xi}}\right)\exp\left\{i\frac{5\pi}{4}\right\} \tag{12}$$

and

$$\dot{Z}(t) = i\lambda A_-\sqrt{\xi}\exp\left\{-\frac{i}{4}\left(\xi t^2 - 2i\Upsilon t - \frac{\Upsilon^2}{4\xi}\right)\right\} \times D_{-i\lambda-1}\left(t\sqrt{\xi} - i\frac{\Upsilon}{\sqrt{\xi}}\right)\exp\left\{i\frac{5\pi}{4}\right\}. \tag{13}$$