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The Boltzmann–Langevin approach: A simple quantum-mechanical derivation

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ABSTRACT

We present a simple quantum-mechanical derivation of correlation function of Langevin sources in the semiclassical Boltzmann–Langevin equation. The specific case of electron–phonon scattering is considered. It is shown that the assumption of weak scattering leads to the Poisson nature of the scattering fluxes.

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1. Introduction

Nonequilibrium electrical noise in mesoscopic systems was always the subject of primary interest for Markus Büttiker. The famous Landauer–Büttiker formula for the shot noise in quantum-coherent conductors became a cornerstone of modern theory of fluctuations [1]. This formula was successfully applied to the calculations of shot noise in different mesoscopic systems with noninteracting electrons ranging from double-barrier resonant tunnel diodes [2] to quantum-coherent metallic diffusive wires [3]. However this method has difficulty in describing interacting electrons or systems with dephasing. To circumvent it, one has to introduce dephasing probes [4,5], i.e. fictitious probes with voltages chosen such that they do not affect the electrical current but allow a replacement of quantum-coherent electrons in the conductor by electrons from reservoirs with a random phase. Yet the properties of the dephasing probes have to be somehow related to the rate of actual microscopic scattering processes.

An alternative method for calculating the electrical noise in conductors in the limit of a large number of quantum channels is the Boltzmann–Langevin approach proposed by Kogan and Shul'man in 1969 [6]. In this approach, the fluctuations of current and any other observable quantities are expressed in terms of the fluctuations of semiclassical distribution function, which obey the Boltzmann equation with a Langevin source in the right-hand side. This method appeared to be very efficient when calculating the hot-electron noise in diffusive metallic wires [7,8], frequency-dependent shot noise in metallic structures in the presence of external screening gates [9,10], and even the noise in hybrid superconductor – normal-metal systems at voltages much higher than the Thouless energy [11]. More recently, it was extended to the case of spin-flip scattering in ferromagnetic spin valves [12] and applied to Coulomb drag in clean double-layer systems [13]

The key point in the Boltzmann–Langevin approach is the derivation of the correlation function of Langevin sources. Kogan and Shul'man derived it assuming that the noise arises due to the randomness of electron scattering by impurities and phonons. It was also assumed that all scattering events are independent, hence the scattering of electrons between a pair of states at a given space point presents a Poisson process, whose spectral density is proportional to its average rate.

Surprisingly, there were few attempts to derive the correlation function of Langevin sources directly from quantum-mechanical principles. In paper [14], this correlation function was calculated using a sophisticated extension of Keldysh diagrammatic technique, which involved time-ordering on a four-branch temporal contour. The current paper presents a much simpler quantum-mechanical derivation of this quantity, which does not require a diagrammatic technique.

2. The general expression

The standard semiclassical distribution function of electrons $n_{\mathbf{p}}(\mathbf{r}t)$ presents the statistical average of the number of electrons in an element of phase space $\Delta p^3 \times \Delta x^3$ divided by the number of quantum states in this element $\Delta N = \Delta p^3\, \Delta x^3/(2\pi\hbar)^3$, which is centered at point $(\mathbf{p}, \mathbf{r})$. This implies that the statistical averaging is performed on top of the coarse-grained averaging [15]. Once the distribution function is known, one may easily calculate different measurable quantities like charge or current density as linear functionals of it. It can be shown in many different ways [15–17]

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that this distribution function obeys the well-known Boltzmann equation:

$$\frac{\partial \bar{n}_{\mathbf{p}}}{\partial t} + \mathbf{v}\frac{\partial \bar{n}_{\mathbf{p}}}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial \bar{n}_{\mathbf{p}}}{\partial \mathbf{p}} = I_{col}\{\bar{n}_{\mathbf{p}}\}, \tag{1}$$

where I_{col} is the collision integral that accounts for the electron scattering by impurities and phonons or electron–electron scattering. This equation is valid provided that the semiclassical approximation holds. First, the characteristic length of spatial variation of $n_{\mathbf{p}}$ must be larger than the microscopic scale responsible for the scattering $\hbar/p_c$, where p_c is the characteristic momentum of an electron. Second, both the characteristic time of its variation and the inverse scattering rate must be larger than the characteristic time of the collision with impurity or phonon $\hbar/\varepsilon_c$, where ε_c is the characteristic energy of an electron.

Along with the average distribution function, one may also be interested in the correlation function of its fluctuations $\langle \delta n_{\mathbf{p}_1}(\mathbf{r}_1 t_1)\delta n_{\mathbf{p}_2}(\mathbf{r}_2 t_2)\rangle$, where the fluctuations $\delta n_{\mathbf{p}_i} = n_{\mathbf{p}_i} - \bar{n}_{\mathbf{p}_i}$ are only coarse-grained-averaged, and the statistical averaging is applied to their product [18]. This quantity immediately gives the correlation functions of different observables. To calculate it, let us take a closer look at Eq. (1). Apart from the time derivative, the terms in the left-hand side describe the deterministic motion of electrons in the phase space due to smooth spatial variations of the distribution function and electrical potential. In contrast to this, the collision integral describes quantum-mechanical transitions of electrons between the states with different momentum, which are assumed to be local in space and time. These transitions are random and should be considered as the source of noise if the semiclassical description is used.

As the structure of the Boltzmann equation without the drift terms resembles the equation of motion of the Brownian particle, one may write the corresponding Langevin equation for the distribution function. To this end, $\bar{n}_{\mathbf{p}}$ should be replaced by $n_{\mathbf{p}}$ and a random Langevin source $\delta J^{ext}_{\mathbf{p}}(\mathbf{r}t)$ with zero average should be added [19] to the right-hand side of Eq. (1). As the duration of an electron collision with an impurity or phonon is much smaller than characteristic time of variation of $n_{\mathbf{p}}$, this source may be assumed to be delta correlated in time. Similarly, it should be delta correlated in space because of the local nature of the collisions. The momentum-dependent coefficient of these delta functions may be calculated as follows [20]. Choose an interval of time Δt much longer than the collision time $\hbar/\varepsilon_c$ but so short that the distribution function cannot significantly change during this period. The direct integration of the Boltzmann–Langevin equation over time gives the increment of $n_{\mathbf{p}}$:

$$\Delta n_{\mathbf{p}} \equiv n_{\mathbf{p}}(\mathbf{r}, t+\Delta t) - n_{\mathbf{p}}(\mathbf{r}t) = \int_0^{\Delta t} d\tau\, \delta J^{ext}_{\mathbf{p}}(\mathbf{r}, t+\tau). \tag{2}$$

Hence the correlation function of two such increments is given by a double integral:

$$\langle \Delta n_{\mathbf{p}_1}(\mathbf{r}_1)\Delta n_{\mathbf{p}_2}(\mathbf{r}_2)\rangle = \int_0^{\Delta t} d\tau_1 \int_0^{\Delta t} d\tau_2 \times \langle \delta J^{ext}_{\mathbf{p}_1}(\mathbf{r}_1, t+\tau_1)\delta J^{ext}_{\mathbf{p}_2}(\mathbf{r}_2, t+\tau_2)\rangle. \tag{3}$$

The delta function of $\tau_1 - \tau_2$ eliminates one of the integrations, and the other reduces to a multiplication by Δt. Hence it follows from Eq. (3) that

$$\langle \delta J^{ext}_{\mathbf{p}_1}(\mathbf{r}_1, t_1)\delta J^{ext}_{\mathbf{p}_2}(\mathbf{r}_2, t_2)\rangle = \delta(t_1 - t_2)\delta(\mathbf{r}_1 - \mathbf{r}_2) \times \lim_{\Delta t\to 0}\frac{\langle \Delta n_{\mathbf{p}_1}(\mathbf{r}_1)\Delta n_{\mathbf{p}_2}(\mathbf{r}_1)\rangle}{\Delta t}. \tag{4}$$

This is our basic formula for calculating the correlation function of Langevin sources.

3. Equations of motion for the operators

To carry out the calculations to the end, we have to calculate the ratio in Eq. (4) using quantum mechanics. Consider the particular case of electron–phonon scattering in an elementary volume of size Δx much smaller than the characteristic length at which the average distribution function or the electrical potential essentially changes but much larger than $\hbar/p_c$. This allows us to describe the scattering with a locally uniform Hamiltonian. The Hamiltonian of the system $\hat{H} = \hat{H}_0 + \hat{V}$ is the sum of the non-interacting part

$$\hat{H}_0 = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} \hat{a}^+_{\mathbf{p}}\hat{a}_{\mathbf{p}} + \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} \hat{b}^+_{\mathbf{k}}\hat{b}_{\mathbf{k}} \tag{5}$$

and the part describing the electron–phonon interaction

$$\hat{V} = \sum_{\mathbf{p},\mathbf{k}} (V_{\mathbf{k}}\hat{b}_{\mathbf{k}} + V^*_{-\mathbf{k}}\hat{b}^+_{-\mathbf{k}})\hat{a}^+_{\mathbf{p}+\hbar\mathbf{k}}\hat{a}_{\mathbf{p}}, \tag{6}$$

where $V_{\mathbf{k}}$ are the matrix elements of electron–phonon interaction, and $\hat{a}_{\mathbf{p}}$ and $\hat{b}_{\mathbf{k}}$ are the annihilation operators for electrons and phonons, respectively. The time-dependent occupation-number operator $\hat{n}_{\mathbf{p}} = \hat{a}^+_{\mathbf{p}}\hat{a}_{\mathbf{p}}$ obeys the Heisenberg equation [21]:

$$\frac{d\hat{n}_{\mathbf{p}}}{dt} = \frac{i}{\hbar}[\hat{H}, \hat{n}_{\mathbf{p}}(t)]. \tag{7}$$

As $[\hat{n}_{\mathbf{p}}, \hat{H}_0] = 0$, the time dependence of this operator is determined only by the weak electron–phonon coupling and may be considered as slow. Therefore Eq. (7) may be solved by iterations in $\hat{V}$. To this end, we perform a unitary transformation of all operators:

$$\tilde{A}(t,\tau) = e^{-(i/\hbar)\hat{H}_0(t)\tau}\hat{A}(t)e^{(i/\hbar)\hat{H}_0(t)\tau}, \tag{8}$$

which brings Eq. (7) to the form

$$\frac{d\tilde{n}_{\mathbf{p}}}{d\tau} = \frac{i}{\hbar}[\tilde{V}(t,\tau), \tilde{n}_{\mathbf{p}}]. \tag{9}$$

If the time interval Δt is much shorter than the relaxation time of the distribution function due to the collisions, the increment of $\tilde{n}_{\mathbf{p}}$ may be calculated to the second order in $\tilde{V}$, and then the transformation inverse to Eq. (8) gives

$$\Delta\hat{n}_{\mathbf{p}} \equiv \hat{n}_{\mathbf{p}}(t+\Delta t) - \hat{n}_{\mathbf{p}}(t) = \frac{i}{\hbar}\int_0^{\Delta t} d\tau[\tilde{V}(t,\tau-\Delta t), \hat{n}_{\mathbf{p}}(t)] - \frac{1}{\hbar^2}\int_0^{\Delta t} d\tau' \int_0^{\tau'} d\tau'' \times [\tilde{V}(t,\tau'-\Delta t), [\tilde{V}(t,\tau''-\Delta t), \hat{n}_{\mathbf{p}}(t)]], \tag{10}$$

where

$$\tilde{V}(t,\tau) = \sum_{\mathbf{p},\mathbf{k}} (V_{\mathbf{k}}e^{i\omega_{\mathbf{k}}\tau}\hat{b}_{\mathbf{k}} + V^*_{-\mathbf{k}}e^{-i\omega_{-\mathbf{k}}\tau}\hat{b}^+_{-\mathbf{k}}) \times e^{(i/\hbar)(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}+\hbar\mathbf{k}})\tau}\,\hat{a}^+_{\mathbf{p}+\hbar\mathbf{k}}\hat{a}_{\mathbf{p}} \tag{11}$$

and all fermionic and bosonic operators are taken at time t. The density matrix of the system is assumed to be factorized into the electron and phonon parts, which are diagonal in the same representation as $\hat{H}_0$. Therefore upon averaging Eq. (10), the first summand vanishes and in the second summand, only diagonal terms are left. The average products of four fermionic operators are decoupled into products of pair averages, e.g. $\langle \hat{a}^+_{\mathbf{p}+\hbar\mathbf{k}}\hat{a}_{\mathbf{p}}\,\hat{a}^+_{\mathbf{q}-\hbar\mathbf{k}}\hat{a}_{\mathbf{q}}\rangle = \delta_{\mathbf{p}+\hbar\mathbf{k},\mathbf{q}}\hat{n}_{\mathbf{q}}(1-n_{\mathbf{p}})$. As a result, all the arguments of exponents are proportional to the difference $\tau' - \tau''$, and

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