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Semiclassical theory of persistent current fluctuations in ballistic chaotic rings

Piet W. Brouwer [a,*], Jeroen Danon [b]

[a] Dahlem Center for Complex Quantum Systems and Physics Department, Freie Universität Berlin, Arnimallee 14, 14195 Berlin, Germany
[b] Niels Bohr International Academy, and the Center for Quantum Devices, Niels Bohr Institute, University of Copenhagen, 2100 Copenhagen, Denmark

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ABSTRACT

The persistent current in a mesoscopic ring has a Gaussian distribution with small non-Gaussian corrections. Here we report a semiclassical calculation of the leading non-Gaussian correction, which is described by the three-point correlation function. The semiclassical approach is applicable to systems in which the electron dynamics is ballistic and chaotic, and includes the dependence on the Ehrenfest time. At small but finite Ehrenfest times, the non-Gaussian fluctuations are enhanced with respect to the limit of zero Ehrenfest time.

1. Introduction

The fact that application of a magnetic field induces an equilibrium charge current is at the basis of the Landau diamagnetic magnetic response of metals [1]. For conducting rings threaded by a magnetic flux, this orbital magnetic response takes the form of a current around the ring, whereas the sign of the response may be diamagnetic as well as paramagnetic [2]. The recognition by Büttiker, Imry, and Landauer that this so-called "persistent current" continues to exist in the presence of elastic impurity scattering [3] and, hence, should be observable in realistic metal samples, initiated a surge in theoretical and experimental work on this paradigmatic mesoscopic phenomenon in the mid-1980s and 1990s [4]. Two recent experiments have revived the interest in persistent currents [5–7]. The magnitude of the measured mean square current is in excellent agreement with the original theoretical predictions for disordered metal rings [8,9]. Earlier experiments had confirmed the existence of the persistent currents [10,11], but a quantitative verification of the theoretical estimates was not possible.

Whereas disorder is unavoidable in metal rings, persistent currents were also investigated in semiconductor heterostructures, for which the electron motion is ballistic [12]. The most pronounced difference between ballistic and disordered-diffusive rings is the possible existence of short periodic electron trajectories in the former, for which the persistent current essentially follows the behavior of ideal one-dimensional rings without potential scattering [13]. Such short trajectories may dominate the magnetic response, even if the classical dynamics in the ballistic conductor is chaotic [14–17].

An interesting case arises if the ballistic conductor has a chaotic classical dynamics, but without short periodic trajectories encircling the magnetic flux [18]. Examples of such a situation are, *e.g.*, a ballistic ring with disc-like scatterers, referred to as a "Lorentz gas", or a collection of chaotic cavities arranged in a ring. Without short periodic trajectories, differences between the ballistic chaotic conductor and its disordered counterpart are much more subtle, related to the "Ehrenfest time" [19]:

$$\tau_{\mathrm{E}} = \frac{1}{\lambda} \ln kL, \tag{1}$$

where λ is the Lyapunov exponent of the classical dynamics, k is the wavenumber, and L a characteristic classical length scale. Being the time required for two classical trajectories a quantum separation $1/k$ apart to acquire a classical separation L under the influence of the chaotic classical dynamics, τ_{E} characterizes the threshold between classical-deterministic and quantum-stochastic dynamics in ballistic structures. Ehrenfest-time-related effects have been considered for equilibrium properties of chaotic quantum dots [20–23], and for quantum transport in open systems [19,24–34], but not for persistent currents in a ring geometry.

In the present paper we report a study of the Ehrenfest-time dependence of the mesoscopic fluctuations of the persistent current in ballistic rings in which the classical electron motion is chaotic and, after appropriate coarse graining, diffusive. We consider a grand canonical ensemble, and assume that time-reversal symmetry in the ring is broken by an applied magnetic field. In a ballistic ring, mesoscopic fluctuations of the persistent current are induced by variations of the chemical potential μ; no disorder average is taken. Differences between ballistic–chaotic conductors and their disordered counterparts appear through a dependence on the Ehrenfest time τ_{E} for the ballistic–chaotic case, whereas τ_{E} plays no role in the case of a

* Corresponding author.
E-mail address: brouwer@zedat.fu-berlin.de (P.W. Brouwer).

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disordered conductor. As we show below, no τ_E – dependence is found on the level of the two-point correlation function $\langle I(\phi_1)I(\phi_2)\rangle$ of the current distribution; only the connected three-point correlation function $K(\phi_1, \phi_2, \phi_3) = \langle I(\phi_1)I(\phi_2)I(\phi_3)\rangle_c$, which describes deviations from the Gaussian distribution, shows a dependence on the Ehrenfest time in the case of a ballistic conductor. (Here ϕ is the flux threading the ring, in units of the flux quantum hc/e; the subscript "c" refers to the 'connected average', $\langle abc\rangle_c = \langle abc\rangle - \langle ab\rangle\langle c\rangle - \langle bc\rangle\langle a\rangle - \langle ca\rangle\langle b\rangle + 2\langle a\rangle\langle b\rangle\langle c\rangle$.)

In Section 2 we describe the starting point of our theoretical approach, Gutzwiller's trace formula, and the semiclassical approximation. A calculation of the two-point correlation function is presented in Section 3, and the three-point correlator is discussed in Sections 4 and 5. We conclude in Section 6.

2. Persistent current from Gutzwiller's trace formula

Starting point of our calculation of the persistent current I is the thermodynamic relation

$$I = -\frac{e}{h}\frac{\partial\Omega}{\partial\phi}, \tag{2}$$

where the thermodynamic potential at temperature T and chemical potential μ

$$\Omega = -T\int d\varepsilon\, \ln(1 + e^{-(\varepsilon-\mu)/T})\nu(\varepsilon) \tag{3}$$

are expressed as an integral of the density of states $\nu(\varepsilon)$. Following previous works on persistent currents in ballistic chaotic conductors [14–17], we use the Gutzwiller trace formula [35] to express the fluctuating contribution to the density of states as a sum over periodic orbits α on the energy shell [36]:

$$\nu(\varepsilon) = \frac{1}{\pi\hbar} Re \sum_\alpha A_\alpha t_\alpha^0 e^{i\mathcal{S}_\alpha(\varepsilon)/\hbar}. \tag{4}$$

In this expression, the label α represents a periodic orbit with primitive period t_α^0 and period $t_\alpha = mt_\alpha^0$, where m is the repetition number. Further $\mathcal{S}_\alpha(\varepsilon)$ is the classical action of the orbit α and A_α the stability amplitude of the orbit:

$$A_\alpha = [\det((M_\alpha^0)^m - 1)]^{-1/2} \tag{5}$$

where M_α^0 is the stability matrix of the primitive orbit α [36].

We now specialize to a two-dimensional system threaded by a flux $\Phi = \phi hc/e$. Considering energies ε near the chemical potential μ, the action $\mathcal{S}_\alpha(\varepsilon, \phi)$ can be written as

$$\mathcal{S}_\alpha(\varepsilon, \phi) = \mathcal{S}_\alpha(\mu, 0) + 2\pi\phi\hbar n_\alpha + (\varepsilon - \mu)t_\alpha, \tag{6}$$

where n_α is the winding number of the trajectory α. Below we will write S_α as short-hand notation for $\mathcal{S}_\alpha(\mu, 0)$. Substituting the Gutzwiller trace formula for the density of states ν, taking the derivative to ϕ, and performing the integration over ε, one finds [18]

$$I = -\frac{ie}{2\pi\hbar}\sum_\alpha \frac{n_\alpha \pi T t_\alpha^0}{t_\alpha \sinh(\pi t_\alpha T/\hbar)} \times (A_\alpha e^{(i/\hbar)\mathcal{S}_\alpha + 2\pi i n_\alpha\phi} - A_\alpha^* e^{-(i/\hbar)\mathcal{S}_\alpha - 2\pi i n_\alpha\phi}). \tag{7}$$

Upon separating the current into Fourier components,

$$I = \sum_n I_n e^{2\pi i n\phi}, \tag{8}$$

with $I_n = I_{-n}^*$, one then arrives at the result

$$I_n = -\frac{ien}{2\pi\hbar}\sum_\alpha \frac{\pi T t_\alpha^0}{t_\alpha \sinh(\pi t_\alpha T/\hbar)} \times (A_\alpha e^{i\mathcal{S}_\alpha/\hbar}\delta_{n_\alpha,n} + A_\alpha^* e^{-i\mathcal{S}_\alpha/\hbar}\delta_{n_\alpha,-n}). \tag{9}$$

3. Mean square current

We now calculate the mean square $\langle I_n I_{-n}\rangle$ for the case that time-reversal symmetry in the ring is broken by an applied magnetic field. The leading contribution to $\langle I_n I_{-n}\rangle$ comes from diagonal contributions:

$$\langle I_n I_{-n}\rangle = \frac{2e^2n^2}{(2\pi\hbar)^2} \times \sum_\alpha \frac{(\pi T)^2(t_\alpha^0/t_\alpha)^2}{\sinh^2(\pi t_\alpha T/\hbar)}|A_\alpha|^2\delta_{n_\alpha,n}. \tag{10}$$

The factor two in the numerator comes from the two terms in Eq. (9), which give equal contributions to $\langle I_n I_{-n}\rangle$.

In order to perform the trajectory sum in Eq. (10), we use a method proposed by Argaman et al. [37]. The summation over classical trajectories is expressed as an integral over the energy shell Q. Introducing a phase space coordinate $\boldsymbol{\mu}$, and denoting with $\boldsymbol{\mu}(t)$ the phase space coordinate obtained by following the classical time evolution for a time t, starting at $\boldsymbol{\mu}$, one has

$$\sum_\alpha t_\alpha^0|A_\alpha|^2\delta_{n_\alpha,n}\delta(t - t_\alpha) = \int_Q d\boldsymbol{\mu}\delta(\boldsymbol{\mu}(t) - \boldsymbol{\mu})\delta_{n(\boldsymbol{\mu},t),n}, \tag{11}$$

where $n(\boldsymbol{\mu}, t)$ is the number of times the trajectory starting at the phase space point μ winds around the flux in the time t. The factor t_α^0 arises, because each trajectory is weighted by a factor t_α^0 upon performing the phase space integration [36]. Upon identifying

$$\delta(\boldsymbol{\mu}(t) - \boldsymbol{\mu})\delta_{n(\boldsymbol{\mu},t),n} = p(\boldsymbol{\mu}, \boldsymbol{\mu}, t|n) \tag{12}$$

as the classical probability density that a particle starting at phase space point $\boldsymbol{\mu}$ is found at the same phase space point at time t, while having passed n times around the flux, we conclude that

$$\langle I_n I_{-n}\rangle = \frac{e^2n^2}{2\pi^2\hbar^2}\int dt \frac{(\pi T)^2}{t\sinh^2(\pi tT/\hbar)} \times \int d\boldsymbol{\mu}\, p(\boldsymbol{\mu}, \boldsymbol{\mu}, t|n). \tag{13}$$

Here we neglected the contribution from orbit repetitions, which is a standard approximation in this field, since the non-primitive orbits at a given period are exponentially outnumbered by primitive orbits with the same period.

For a two-dimensional ring of circumference L with diffusive electron dynamics, one has

$$p(\boldsymbol{\mu}, \boldsymbol{\mu}, t|n) = \frac{L}{Q}\frac{e^{-(nL)^2/4Dt}}{\sqrt{4\pi Dt}}, \tag{14}$$

where $Q = 2\pi\hbar\tau_H$ is the volume of the energy shell, τ_H being the Heisenberg time, and D the classical diffusion constant. One then arrives at the result

$$\langle I_n I_{-n}\rangle = \frac{e^2n^2}{2\pi^2\hbar^2}\int dt \frac{(\pi T)^2}{t\sinh^2(\pi tT/\hbar)} \times \sqrt{\frac{\tau_L}{4\pi t}}e^{-\tau_L n^2/4t}, \tag{15}$$

where