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Measuring the Luttinger liquid parameter with shot noise

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HIGHLIGHTS

- We explore the low-frequency noise of interacting electrons in one dimension.
- The system is driven out of equilibrium by a QPC with an applied voltage.
- A second QPC serves to explore the statistics of outgoing electrons.
- Low-frequency noise in such a setup allows to measure the Luttinger liquid constant.

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ABSTRACT

We explore the low-frequency noise of interacting electrons in a one-dimensional structure (quantum wire or interaction-coupled edge states) with counterpropagating modes, assuming a single channel in each direction. The system is driven out of equilibrium by a quantum point contact (QPC) with an applied voltage, which induces a double-step energy distribution of incoming electrons on one side of the device. A second QPC serves to explore the statistics of outgoing electrons. We show that measurement of a low-frequency noise in such a setup allows one to extract the Luttinger liquid constant K which is the key parameter characterizing an interacting 1D system. We evaluate the dependence of the zero-frequency noise on K and on parameters of both QPCs (transparencies and voltages).

1. Introduction

The physics of interacting electrons in one dimension (1D) is profoundly different from that in higher dimensions. It is well known that the correspondence between interacting electrons and free fermionic quasiparticles, which is in the core of Landau's Fermi-liquid theory, breaks down in 1D. The resulting strongly correlated state, known as the Luttinger liquid (LL), cannot be treated by conventional Fermi-liquid methods. Fortunately, there exists an extremely powerful approach to the problem, the bosonization technique [1–6]. It describes the low-energy sector of the theory in terms of density fluctuations, which are, under the simplest circumstances, non-interacting bosons.

A key parameter invoked in the bosonization description of a LL state is the interaction constant K. This dimensionless parameter gives an effective measure of the strength of the interaction between the electrons, with $K=1$ corresponding to a non-interacting Fermi gas, $K<1$ to repulsion, and $K>1$ to attraction. The LL constant K controls the behavior of various physical properties of the system [3], including, e.g., the scaling of the tunneling density of states away from the wire ends (TDOS) [7], $\nu(\epsilon) \propto |\epsilon|^{(1-K)^2/2K}$, the temperature-dependence of the conductance through a tunnel barrier in a Luttinger liquid [8], $G(T) \propto T^{2(1-K)/K}$, and the temperature scaling of the conductivity of a disordered interacting wire [9]. There exists by now a rich variety of experimental realizations of LLs with fermionic constituent particles, including semiconductor, metallic, and polymer nanowires [10], carbon nanotubes [11], edge states of 2D topological insulators [12], and cold-atom systems [13].

Further, edges of quantum Hall systems [14–16] give rise to chiral LLs with only one propagation direction. When two such edges with opposite chirality are coupled by interaction, an artificial "wire" emerges [17,18]. Properties of LL structures are probed in a growing number of sophisticated experiments, in particular under strongly non-equilibrium conditions. A quantitative

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interpretation of experimental findings requires the knowledge of the LL parameter K of the studied system by an additional independent measurement of K.

Let us consider a typical experimental setup where a 1D conductor is connected to the outside world by leads. One possibility to access the value of K is to measure the power-law behavior of the TDOS by exploring the tunneling into the Luttinger liquid. This requires, however, an introduction of an additional probe to the interacting wire and is not simple experimentally. In addition, the tunneling characteristics may be affected by the interaction of the wire with the environment [19]. We can ask if it is possible to infer K considering the LL as a “black box” (in the spirit of scattering theory of electronic conduction [20]) and performing electrical measurements in the leads alone. One could naively expect that the interaction inside the system modifies the conductance of the wire, thus providing a direct experimental way to measure K. It is not correct, however. It is well known [21–24] that, due to absence of fermionic backscattering in a clean LL, its DC conductance is given by the interaction-independent value e^2/h. Moreover, while under generic non-equilibrium conditions the distribution function of the electrons that have passed the interacting part of the system depends on the interaction strength [25], the zero-frequency full counting statistics of the charge transferred through the system [26] is insensitive to the interaction. On the other hand, the non-equilibrium noise [27] and the full counting statistics [26] at high frequencies (of the order of or larger than the inverse flight-time through the system) do depend on the interaction strength but they are challenging to measure experimentally [28].

In this work we show that the interaction in a LL wire can, however, be probed by low-frequency charge noise measurements provided that the electrons emerging from the LL are mixed (via scattering at an additional quantum point contact, QPC) with electrons coming from an independent reservoir. A similar approach was proposed recently to probe the (pseudo-)spin-charge separation in systems of co-propagating channels [29].

The structure of the paper is as follows. In Section 2 we introduce a device, consisting of 4 sources (SL, SR, S1, S2) and 2 drains D1 and D2 that are connected by two point contacts characterized by transmission and reflection coefficients t^2 and r^2 (see Fig. 1). The system is driven out of equilibrium by an

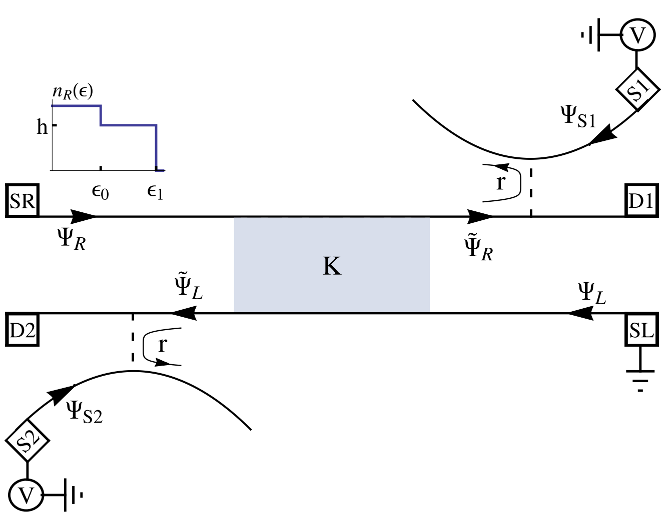

Fig. 1. Setup. Incoming R-electrons have a non-equilibrium double-step energy distribution (1) characterized by the step width $U = \epsilon_1 - \epsilon_0$ and height h. This distribution may be prepared by means of a QPC0 (not shown). The parameter h is given in this case by the transmission probability of the QPC0, while the parameter U is the QPC0 voltage. Incoming L-electrons, as well as $S1$- and $S2$-electrons are at equilibrium but the distribution of the $S1$ and $S2$ electrons can be tuned with a voltage V.

“injection” of electrons with double-step energy distribution trough the source SR. Such a distribution may be naturally prepared by means of an additional QPC0 (not shown in Fig. 1). The step height h is then given by its transmission coefficient. In Section 3 we calculate the shot noise in drain D2 as a function of the Luttinger liquid parameter K. Section 3.1 is devoted to description of the general formalism while Section 3.2 summarizes the results in the limits of weak (|K − 1|â ª¡ 1) and strong (K â ª¡ 1) interaction for voltage U in SL much larger than the inverse of the flight time τ_l in the interaction region. In Section 3.3 we present few numerical results for generic values of interaction parameter K.

Specifically, Figs. 5 and 6 illustrate the central results of the paper. Fig. 5 demonstrates the dependence of the noise at zero voltage V (at source $S2$) on the parameter h of the double-step distribution (1) of incoming right-moving electrons. The noise attains its maximal value when the initial distribution is particle–hole symmetric ($h{=}0.5$) and the QPC mixing the electrons from the LL wire with those from the source S2 has reflection probability $r^2 = t^2 = 0.5$. The ratio of this maximal noise to the voltage U in SL is a universal function of the LL parameter. Fig. 6 shows that the maximal current noise at drain D2 at zero frequency and zero voltage in source S2 ($\omega{=}0$ and $V{=}0$), which we denote as $\max_{h,r^2,t^2}[S_{D2}(\omega = 0, V = 0)]$, provides a direct access to the value of the LL parameter K. Although we do not have a simple analytic expression for the noise in this situation, the curve is universal and our numerical results can be used to determine K.

Section 4 presents the calculation of the noise in drain D1 and Section 5 discusses the situation for attractive interactions. We conclude the paper with a summary section, Section 6.

2. Setup

A setup that we consider in the present paper (and that is particularly relevant in the context of quantum Hall physics, see, e.g., Ref. [18]) is shown in Fig. 1. It includes two counter-propagating electronic (or, more generally, fermionic) modes, right-movers R and left-movers L, interacting over a distance l via a short-range interaction characterized by the LL parameter K. The system is driven out of equilibrium by an “injection” of incoming R-electrons (source SR) with a double-step energy distribution,

$$n_R(\epsilon) = (1 - h)n_0(\epsilon - \epsilon_0) + hn_0(\epsilon - \epsilon_1), \tag{1}$$

Here, $n_0(\epsilon) = \Theta(-\epsilon)$ is the zero-temperature Fermi–Dirac distribution with zero chemical potential and $\epsilon_0 = -hU$, $\epsilon_1 = (1 - h)U$ are the positions of the Fermi edges [30]. The double-step distribution (1) may be naturally prepared by means of a QPC0 (not shown in Fig. 1). The parameter h is given in this case by the transmission probability of the QPC0, while the parameter U is the QPC0 voltage. (We set the electron charge e to unity throughout the paper, restoring it in the final expressions only.) The left-moving mode starts at zero temperature and zero voltage from the source SL. After traversing the interacting part of the wire, right-movers and left-movers are mixed with electrons from sources S1 and S2 (kept at zero temperature and chemical potential V) via scattering at QPCs with transmission (reflection) amplitudes t (r). We are interested in the charge noise at drains D1 and D2

$$S_{D1/D2}(\omega, V) = \int_{\infty}^{\infty} dt \left\langle \left\{ \delta I_{D1/D2}(t), \delta I_{D1/D2}(0) \right\} \right\rangle e^{-i\omega t}, \tag{2}$$

where δI_{Di} is the fluctuating part of the current operator at the drain i, and curly brackets denote the anticommutator. The second argument of $S_{D1/D2}$ in Eq. (2) emphasizes the dependence of noise on the voltage V applied to the sources S1 and S2.

Closing this section let us specify the hierarchy of the energy scales in our problem. First, we assume throughout the paper that