Physica E 57 (2014) 6–11

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

A nonlocal shell theory model for evaluation of thermoelastic damping in the vibration of a double-walled carbon nanotube

M.S. Hoseinzadeh, S.E. Khadem *

Department of Mechanical Engineering, Tarbiat Modares University, P.O. Box 14115-177, Tehran, Iran

HIGHLIGHTS

- A double-walled carbon nanotube is modeled as two individual cylindrical thin shells.
- Nonlocal shell theory is used to investigate thermoelastic vibration behavior of DWCNTs.
- Small-size effects decrease natural frequencies and increase thermoelastic damping compared to the local model.
- For upper coaxial frequency modes, the small-size effect is more profound.

ARTICLE INFO

Article history:
Received 25 February 2013
Received in revised form
7 July 2013
Accepted 9 October 2013
Available online 22 October 2013

Keywords:
Thermoelastic damping
Nonlocal elasticity theory
Free vibration
Double-walled carbon nanotubes
Cylindrical shell model

ABSTRACT

Thermoelastic damping (TED) is a major factor of dissipating energy in the vibration control of nanodevices. On the other hand, application of classic theory in the study of nanostructures is not reasonable. In this paper, a model based on nonlocal shell theory, accounting for the small-scale effects, is used to investigate thermoelastic vibration behavior and damping of double-walled carbon nanotubes (DWCNTs) with simply supported boundary conditions. The inner and outer carbon nanotubes are considered as two individual thin shells. The set of general thermoelastic coupled equations are numerically solved. The results show that the small-scale effects decrease natural frequencies and increase thermoelastic damping compared to the local model, especially for the coaxial frequency and large circumferential wave numbers. The numerical results also show that when the radius of nanotubes rises, the influence of small-size effect on natural frequencies and thermoelastic damping drops dramatically.

1. Introduction

Carbon nanotubes exhibit superior mechanical properties and appealing thermal and electrical conductivity over any known material. The nanometer-size of carbon nanotubes holds substantial promise for new applications in nanobiological devices and nanomechanical systems [1–3]. Therefore, many experimental and theoretical investigations have been carried out to understand the properties and behavior of the CNTs [4–6].

Continuum models especially elastic shell models are relatively simple and cost-effective as compared to experiments and molecular dynamic simulations.

Due to the multi-walled nature of CNTs, there are forces between the carbon atoms that make up the cylindrical walls. These forces, called van der Waals forces, cannot be neglected due to their significance at this molecular scale.

The sensitivity of a mechanical oscillator, which usually operates at its harmonic resonance frequency, depends mainly on the quality factor Q (the inverse of the energy dissipation) of the oscillator. By entering the nano-scale the quality factor is reduced. Thus, the investigation of different methods of energy dissipation becomes essential. Thermoelastic damping is the main parameter in the reduction of the quality factor in nano-oscillator. Zener [7] studied the theory of thermoelasticity for the first time. Recent researches in the analysis of thermoelastic damping belong to continuous models in beams, plates and shells [8–11]. But in these studies, the shell model has not been used as frequently as the two other models. Nayfeh and Younis [8] by considering microplates under electrostatic actuation studied the TED for variable voltage, temperature and thickness of a microplate. Their results show that electrostatic forces increase TED. Hajnayeb et al. [9] utilized a double-elastic beam model for thermoelastic vibrations of double-walled carbon nanotubes under electrostatic actuation and investigated the effect of the geometrical properties and applied DC voltage on the quality factor of the nanotubes. They expressed that shorter nanotubes under higher voltage have greater values of TED.

* Corresponding author. Tel.: +98 2182883388.
E-mail address: khadem@modares.ac.ir (S.E. Khadem).

1386-9477/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.physe.2013.10.009

Taking advantage of shell models for short tubes, Lu et al. [10] studied the TED in cylindrical shell structures and applied the results to a single-walled nanotube as an example. Hoseinzadeh and Khadem [11] by considering intermolecular interaction and initial axial stresses, used a shell model to examine TED for DWCNT with various boundary conditions. It is apparent that in all the studies on TED for continuous models up to now, the classical elastic theory has been used, which is on the assumption that carbon nanotubes are solid and homogenized while the material properties at the nano-scale are size dependent. Thus, the small length scale effect needs to be considered for a better prediction of the mechanical behavior of the nano-materials.

The essence of the nonlocal elasticity theory developed by Eringen [12] indicates that the stress state at a given reference point is a function of the strain field at any point in the body.

Wang et al. [13], by using the nonlocal beam theory in vibration of nanotubes, showed that nonlocal effect reduces natural frequencies. Also, Lee and Chang [14] indicated that in vibration of nanotubes conveying fluid, small size has a stronger effect at low flow velocities and high mode numbers.

In addition to nonlocal beam models, the CNTs with low length-to-diameter ratios (L/R, where L is the length of the nanotube) have been modeled with elastic shell models, where nonlocal shell models have become indispensable, especially when the length-to-radius ratio of the CNTs decreases. Li et al. [15] investigated the vibrational behavior of the multiwalled carbon nanotubes embedded in an elastic medium by a nonlocal shell model. They expressed that when the order of the geometric size of the structures is beyond the nanometer range, the influence from the small scale parameters could be neglected. Arash and Ansari [16], based upon a nonlocal shell, studied the vibration characteristics of single-walled carbon nanotubes with different boundary conditions subjected to initial strains. Their results show that the coefficient of small-size effect depends on boundary conditions.

So, it is certain that there are no studies to examine the influence of nonlocal effect on thermoelastic damping. The primary objective of this paper is to analyze thermoelastic vibration of double-walled carbon nanotubes based on cylindrical shell theory and the role of small-size effect on natural frequencies and thermoelastic damping of vibration of DWCNT.

2. Modeling and formulation

In nonlocal elasticity, the stress at a reference point x is considered to be a functional of the strain field at any point in the body so that the stress state at a reference point in the body is regarded to be dependent not only on the strain state at that point but also on the strain states at all the points throughout the body. For homogeneous and isotropic elastic solids, the constitutive equations of nonlocal elasticity can be written as [15]

$$\sigma_x-(e_0a)^2\nabla^2\sigma_x=\frac{E}{1-\nu^2}(\varepsilon_x+\nu\varepsilon_\theta) \quad (1-a)$$

$$\sigma_\theta-(e_0a)^2\nabla^2\sigma_\theta=\frac{E}{1-\nu^2}(\varepsilon_\theta+\nu\varepsilon_x) \quad (1-b)$$

$$\sigma_{\theta x}-(e_0a)^2\nabla^2\sigma_{\theta x}=\frac{E}{2(1+\nu)}\gamma_{x\theta} \quad (1-c)$$

where 'a' is an internal characteristic length (for example: lattice parameter, granular distance), e_0 is a constant appropriate to each material and E, ν, are the elastic modulus and Poisson's ratio, respectively.

Substituting Eq. (1) into the three equations of motion in the cylindrical coordinate system [11] leads to the nonlocal cylindrical shell theory expressed by

$$\frac{\partial(N_{xx}-\tilde{N}_T+(e_0a)^2\nabla^2\tilde{N}_T)}{\partial x}+\frac{1}{R_i}\frac{\partial N_{\theta x}}{\partial\theta}-\rho h(1-(e_0a)^2\nabla^2)\frac{\partial^2u_i}{\partial t^2}=0 \quad (2-a)$$

$$\frac{\partial N_{\theta x}}{\partial x}+\frac{1}{R_i}\frac{\partial(N_{\theta\theta}-\tilde{N}_T+(e_0a)^2\nabla^2\tilde{N}_T)}{\partial\theta}+\frac{1}{R_i}\frac{\partial M_{\theta x}}{\partial x}+\frac{1}{{R_i}^2}\frac{\partial(M_{xx}-\tilde{M}_T+(e_0a)^2\nabla^2\tilde{M}_T)}{\partial\theta}-\rho h(1-(e_0a)^2\nabla^2)\frac{\partial^2v_i}{\partial t^2}=0 \quad (2-b)$$

$$\frac{\partial^2(M_{xx}-\tilde{M}_T+(e_0a)^2\nabla^2\tilde{N}_T)}{\partial x^2}+\frac{1}{{R_i}^2}\frac{\partial^2M_{x\theta}}{\partial x\partial\theta}+\frac{1}{{R_i}^2}\frac{\partial^2(M_{\theta\theta}-\tilde{M}_T+(e_0a)^2\nabla^2\tilde{M}_T)}{\partial\theta^2}-\frac{N_{\theta\theta}-\tilde{N}_T+(e_0a)^2\nabla^2\tilde{N}_T}{R_i}+p_i-(e_0a)^2\nabla^2p_i-\rho h(1-(e_0a)^2\nabla^2)\frac{\partial^2w_i}{\partial t^2}=0 \quad (2-c)$$

where u_i v_i and w_i ($i=1, 2, \ldots, N$) are the longitudinal, circumferential and radial displacement components of the ith tube, t is the time, ρ is the mass density, h is the thickness, p_i is the pressure exerted on the tube due to vdW interaction between walls, N_{ij} is the resultant of the membrane force, and M_{ij} is the resultant of the bending moment.

It is assumed that attractive vdW force is negative and the repulsive vdW interaction is positive. Thus, the pressure due to vdW interaction can be expressed as [17]

$$p_i(x,\theta)=\sum_{i=1}^{N}C_{ij}(w_i-w_j) \quad (3)$$

in which C_{ij} is the vdW interaction coefficient and depends on R_i.

The thermal membrane force $\tilde{N}_T$ and bending moment $\tilde{M}_T$ can be obtained as

$$\tilde{N}_T=\frac{E\alpha_t}{1-\mu}\int_{-h/2}^{h/2}(T-T_0)\,dz \quad (4-a)$$

$$\tilde{M}_T=\frac{E\alpha_t}{1-\mu}\int_{-h/2}^{h/2}(T-T_0)z\,dz \quad (4-b)$$

where α_t is the coefficient of thermal expansion.

For transverse vibrations, the general governing equations given above can be simplified with Donnell–Mushtari–Vlasov approach. Based on this simplification

$$\varepsilon_{xx}=\varepsilon_{xx}^0+z\kappa_{xx}=\left(\frac{\partial u}{\partial x}\right)+z\left(-\frac{\partial^2w}{\partial x^2}\right) \quad (5-a)$$

$$\varepsilon_{\theta\theta}=\varepsilon_{\theta\theta}^0+z\kappa_{\theta\theta}=\left(\frac{1}{R}\frac{\partial v}{\partial\theta}+\frac{w}{R}\right)+z\left(-\frac{1}{R^2}\frac{\partial^2w}{\partial\theta^2}\right) \quad (5-b)$$

$$\varepsilon_{x\theta}=\varepsilon_{x\theta}^0+z\kappa_{x\theta}=\left(\frac{1}{R}\frac{\partial u}{\partial\theta}+\frac{\partial v}{\partial\theta}\right)+z\left(-\frac{2}{R}\frac{\partial^2w}{\partial x\partial\theta}\right) \quad (5-c)$$

and the first two equations in Eq. (2) can be reduced to

$$\frac{\partial(N_{xx}-\tilde{N}_T+(e_0a)^2\nabla^2\tilde{N}_T)}{\partial x}+\frac{1}{R_i}\frac{\partial N_{\theta x}}{\partial\theta}=0 \quad (6-a)$$

$$\frac{\partial N_{\theta x}}{\partial x}+\frac{1}{R_i}\frac{\partial(N_{\theta\theta}-\tilde{N}_T+(e_0a)^2\nabla^2\tilde{N}_T)}{\partial\theta}=0 \quad (6-b)$$

Now, introducing a function ϕ related to N_{ij}

$$N_{xx}-\tilde{N}_T+(e_0a)^2\nabla^2\tilde{N}_T=\frac{1}{R^2}\frac{\partial^2\phi}{\partial\theta^2} \quad (7-a)$$

$$N_{\theta\theta}-\tilde{N}_T+(e_0a)^2\nabla^2\tilde{N}_T=\frac{\partial^2\phi}{\partial x^2} \quad (7-b)$$