Physica E 57 (2014) 83–95

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Nonlocal vibration and buckling analysis of single and multi-layered graphene sheets using finite strip method including van der Waals effects

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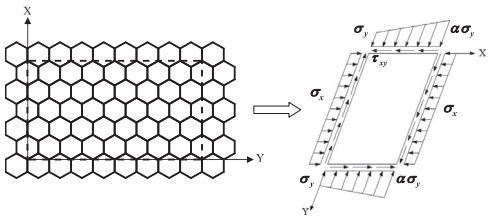
HIGHLIGHTS

- Vibration and buckling of single and multi-layered graphene sheets are considered.
- The finite strip method is applied to study the GS's problems for the first time.
- Nonlocal continuum mechanics is employed to include the small scale-effects.
- Effect of van der Waals forces is considered in the stiffness matrix of the system.
- Contrary to the most of the studies, different boundary conditions are considered.

GRAPHICAL ABSTRACT

The finite strip method is proposed to study the vibration and buckling characteristics of rectangular single and multi-layered graphene sheets, based on the nonlocal classical plate theory taking into account the effects of van der Waals forces between the layers.



ARTICLE INFO

Article history:
Received 31 May 2013
Received in revised form
4 November 2013
Accepted 4 November 2013
Available online 11 November 2013

Keywords:
Graphene sheets
Nonlocal elasticity
Free vibration
Buckling
Van der Waals forces
Finite strip method

ABSTRACT

Detailed studies on the nanoscale vibration and buckling characteristics of rectangular single and multi-layered graphene sheets (SLGSs and MLGSs) are carried out using semi-analytical finite strip method (FSM), based on the classical plate theory (CPT). The displacement functions of the sheets are evaluated using continuous harmonic function series which satisfy the boundary conditions in one direction and a piecewise interpolation polynomial in the other direction. Nonlocal continuum mechanics is employed to derive the differential equation of the system. The weighted residual method is employed to obtain stiffness, stability and mass matrices of the graphene sheets. The effects of van der Waals (vdW) forces which are present as bonding forces between the layers are considered in the stiffness matrix of the system. The analysis of MLGSs is much more complex due to the influence of vdW forces. The mechanical properties of the graphene sheet are assumed in two ways as orthotropic or isotropic materials. A matrix eigenvalue problem is solved to find the natural frequency and critical stress of GSs subjected to different types of in-plane loadings including uniform and non-uniform uniaxial loadings. The accuracy of the proposed model is validated by comparing the results with those reported by the available references. Furthermore, a comprehensive parametric study is performed to investigate the effects of various parameters such as boundary conditions, nonlocal parameter, aspect ratio and the type of loading on the results.

1. Introduction

Invention of carbon nanotubes (CNTs) in 1991 by Iijiima [1] opened a wide space to the scientific community. CNTs have many

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http://dx.doi.org/10.1016/j.physe.2013.11.002

potential applications, due to their great mechanical, chemical, thermal, electrical and electronic characteristics [2]. CNTs are classified into single-walled (SWCNTs) and multi-walled carbon nanotubes (MWCNTs). Graphene, the new miracle material which was introduced in 2004 [3], is also another type of carbon nanomaterials with a two-dimensional structure. It possesses many superior properties such as good flexibility and high thermal and electrical conductivity [4,5]. Similar to CNTs, two groups of graphene sheets (GSs) are considered in the literature, namely single-layered graphene sheets (SLGSs) and multi-layered graphene sheets (MLGSs).

The application of these nanostructures is rapidly growing in different areas such as aerospace, microelectronics, micro electromechanical systems (MEMS), nanoelectromechanical systems (NEMS) and nanocomposites [6–8]. Hence, it is indispensable to recognize and analyze different behaviors of nanostructures. Alongside the experimental works, various analytical and numerical methods have been recently done to achieve this aim. These methods can be classified as atomistic-based methods and continuum-based methods. Atomistic-based methods include the molecular dynamics simulation (MD) [9,10], tight-binding molecular dynamics [11] and density function theory [12]. Because of the difficulties in nanoscale experiments and the expensive computations of atomistic based methods for a large system of atoms, development of appropriate mathematical models based on classical continuum mechanics has been an important issue in the analysis of CNTs and GSs. Different exact and numerical continuum mechanics models have been used by researchers to capture the accurate results. However, owing to their small dimensions, the application of classical continuum models may be debatable in the analysis of nanostructures. Experiments demonstrate that the behavior and mechanical properties of nanomaterials are affected by scale-effect or size-effect. Size-effects are related to long-range inter-atomic interactions and could not be ignored in the analysis of nanostructures. Therefore to predict the accurate results, it is necessary to consider the small scale-effects. One of the most reported size-dependent continuum theories is the nonlocal elasticity theory proposed by Eringen [13–15], in which the forces between atoms and the internal length scale are considered in the formulation. The small-scale effects are captured by assuming that the stress at a reference point is a function of the strain field at every point in the domain. This is unlike classical elasticity theory. The results of nonlocal models are in accordance with atomistic results of lattice dynamics and MD simulations [16]. So far, a large number of nonlocal based studies, focused on static bending, dynamic vibration and stability analysis of nanostructures have been reported. These include analysis of nanobeams [17–20], nanotubes [21–26], nanorods [27], nanorings [28] and nanoplates [29–31].

The popular nanoplates which have recently captured attention of scientific community are SLGSs and MLGSs. GSs could be considered as nanoplates to which the governing equations of different plate theories, taking into account the small scale-effect, could be applied. Moreover, the synthesis of graphene-based nanocomposites has opened a new path in the research topics. Due to these increasing applications, understanding the mechanical behaviors of GSs is essential for their engineering design and manufacture. Different vibration and buckling studies of SLGSs and MLGSs with and without the surrounding elastic medium have been reported in the literature [32–57]. These papers mostly applied the nonlocal elasticity theory and used different methods such as differential quadrature method (DQM) and finite element method (FEM).

The present work addresses the vibration and buckling behavior of single and multi-layered graphene sheets using the finite strip method (FSM). In comparison with other numerical methods such as FEM, the finite strip method, which was first introduced by Cheung [58], provides more efficient formulations for investigation of plate behavior under different loads and boundary conditions. However, hitherto its direct application to the vibration and buckling behavior of rectangular GSs has not been investigated. Consequently, in this paper the semi-analytical FSM is employed to investigate the vibration of SLGSs and MLGSs as well as their buckling behavior when subjected to uniform and non-uniform uniaxial compressive loadings. The van der Waals (vdW) interatomic forces between layers of MLGSs are considered in the derivation of stiffness matrices. Moreover, the present methodology considers the influence of small scale-effects on the vibration and bucking of GSs. Contrary to the majority of the studies, different boundary conditions are considered in this study. Effects of length, aspect ratio, higher modes and nonlocal parameter are considered in the results.

This paper is organized into the following sections. In Section 2, the Eringen's nonlocal theory of elasticity and its application in classical plate theory is overviewed. In Section 3, mathematical formulation of the finite strip method is presented and the method is developed to study the buckling and vibration of graphene sheets. Numerical results and discussions are demonstrated in Section 4 for two types of SLGSs and MLGSs. Finally, some concluding remarks are presented in Section 5.

2. Theory

The aim of this part is to introduce Eringen's nonlocal theory of elasticity and its application in classical plate theory.

2.1. Nonlocal theory

According to the nonlocal elasticity theory [13–15], the relation between stress and strain is expressed as

$$\sigma_{ij}(x) = \int_V \lambda(|x-x'|,\tau) C_{ijkl}\varepsilon_{kl}(x')dV(x') \tag{1}$$

where σ_{ij}, ε_{kl} and C_{ijkl} are the stress, strain and components of the fourth order elasticity tensor, respectively. Also, $\lambda(|x-x'|,\tau)$ is the nonlocal modulus or attenuation function which incorporates the nonlocal effects into constitutive equations, in which $|x-x'|$ represents the Euclidean form of the distance between x and x', and τ is a material constant that depends on the characteristic length ratio l_i/l_e, where l_i is an internal characteristic length (e.g., lattice parameter, granular distance, distance between C–C bonds) and l_e is an external characteristic length (e.g., crack length, wave length). The material constant τ is defined as $\tau = e_0(l_i/l_e)$ and could be evaluated by experimental methods or the molecular dynamics (MD) simulation method. The parameter e_0 extremely depends on the complicated internal microstructures of nanomaterials; it is estimated such that the relations of the nonlocal elasticity model provide satisfied approximation of atomic dispersion curves of plane waves with those of atomic lattice dynamics [48].

An equation in differential form is used as an alternative to Eq. (1), because it is difficult to deal with integral constitutive. This equation which is the basis of all nonlocal constitutive formulations, is expressed as

$$\Gamma\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{2}$$

where Γ denotes the nonlocal operator and is represented by Eringen as

$$\Gamma = 1-\mu\nabla^2 \tag{3}$$

in which $\mu = (e_0 l_i)^2$ and ∇^2 is the Laplacian operator. Therefore the equation of nonlocal elasticity is represented as

$$(1-\mu\nabla^2)\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{4}$$