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On the bending and buckling behaviors of Mindlin nanoplates considering surface energies

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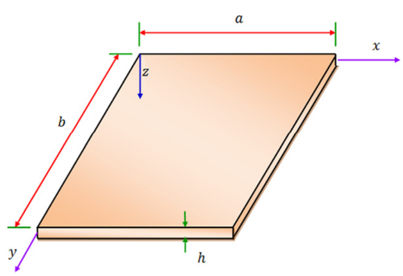
HIGHLIGHTS

- Development of a non-classical continuum model using Gurtin–Murdoch surface theory.
- Study of bending and buckling behaviors of nanoplates with different boundary conditions.
- Surface property is found to be effective in the surface effect on the response of nanoplates.
- Nanoplates with stiffer edge conditions is observed to be less affected by the surface influences.

GRAPHICAL ABSTRACT

A Mindlin plate model is developed to describe the bending and buckling characteristics of nanoplates including the surface stress effect.



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ABSTRACT

Due to the high surface to volume ratio of the nanoscale domain, the surface stress effect is a major concern in the analysis of mechanical response of the nanomaterials and nanostructures. This paper is concerned with the applicability of a continuum model including the surface properties for describing the bending and buckling configuration of the nanoscale plates. The Gurtin–Murdoch surface theory of elasticity is first incorporated into Mindlin's plate theory. Then, the principle of virtual work is applied to derive the size-dependent governing equations along with various boundary conditions. To solve the governing equations, the generalized differential quadrature (GDQ) method is employed. The critical uniaxial and biaxial buckling loads and the maximum deflection of the nanoplate due to a uniform transverse load are calculated in the presence and absence of the surface effects for various edge conditions. It is found that the significance of the surface effects on the response of the nanoplate relies on its size, type of edge support and selected surface constants.

1. Introduction

One of the fastest developing and emerging fields of research which has had a profound effect on the industry, economy and finally the life of human society is nanotechnology. This interdisciplinary technology has paved the way for the next step in the creation of a new generation of materials with supreme properties and devices with enhanced potential application. Nanotechnology includes the production and application of systems in various fields and scales. The structures at nanoscale such as nanoplates, nanobeams and nanotubes can be further integrated into the larger systems. These nanostructures produced by some molecular manipulations are viewed as the essential building blocks for different types of nanosystems and nanodevices [1–4]. When the scale of a structure becomes very small, the ratio of surface area to volume of the structure increases. In this manner, the material properties of the boundary layers of the elastic media turn to be different from those of the bulk materials [5,6]. This is attributed to the fact that the equilibrium requirements of the atoms at or near free surface are not same as those of the atoms in the bulk of

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the material [7]. These extra properties known as the surface energies affect on the physical, mechanical and electrical properties as well as the mechanical response of the nanostructure and can cause interesting behaviors [8–11]. For example, Wang et al. [10] found that the surface buckling may occur in a nanobeam with stiffer surface materials when subjected to pure bending. Fedorchenko et al. [11] showed that, in the presence of surface effects, the elastic modulus of a nanoplate is a size-dependent parameter.

Since the properties of a nanostructure are extremely sensitive to even infinitesimal stimulations, its deformation under different loadings can be of much significance. On the other hand, the development of nanotechnology and then the increasing application of the nanostructures in different fields make the analysis of the mechanical behavior of them an essential issue. To describe the size-dependent behavior of the nanostructures, some researchers used the modern scientific manipulations to do the experimental investigation into the mechanical response at the nanometer scale [12–18]. Also, the atomistic methods as the fundamental approaches capturing the small scale effects were generally applied and provided plentiful results [19–24]. However, these methods face the challenges such as the much difficulty induced by doing controlled experiments at nanoscale or the high computational expenses induced by the atomic modeling in the larger scale structures. Hence, the continuum-based theoretical approach as an efficient computational technique has drawn a great deal of attention from the scientific society. To take the scale effects into account, the classical continuum methods are modified lading to the development of the size-dependent continuum theories namely the surface theory of elasticity [25,26] and the higher-order continuum theory [27,28].

In the surface theory of elasticity pioneered by Gurtin and Murdoch [25,26], the surface layer of a solid is viewed as a mathematical layer of zero thickness having different material properties from those of the underlying bulk perfectly attached by the membrane. Based on this theory, much research has been done in connection with the mechanical behavior of the nanostructures [29–38]. Zhang et al. [29] presented three models of the surface stress on the rectangular cantilever beams and compared them to each other. They investigated the disagreement arisen between the experimental data and the theoretical prediction. He et al. [30] studied the size-dependent deformation of nanofilms on the basis of the continuum mechanics incorporating the surface stress effects. Lu and his co-workers [31] incorporated the surface properties into the generalized Kirchhoff's and Mindlin's plate theories to study the size-dependent static bending, vibration and buckling of the nanoplates. Wang and Feng [32] studied the effects of the surface stresses on nano-sized contact problems and discerned that the surface stress of the nano-indentation strongly influences on both the maximum normal contact stress and the indent depth. Assadi et al. [33] considered a nanoplate under different thermal environments and studied the size-dependent dynamic behavior of it using size and temperature-dependent elastic modulus along with the surface effects. Wang investigated the flexural vibration of fluid conveying nanotubes taking the effects of the surface properties into account [34]. Ansari and Sahmani [35] evaluated the surface stress effects on the displacement profile and the critical buckling load of the nanobeams based on the different beam theories. Fu and Zhang [36] analyzed pull-in phenomenon in the electrically actuated nanobeams considering the surface effects. Based on the Gurtin–Murdoch theory, Ansari and his co-workers [37] investigated the size-dependent pull-in instability of the hydrostatically and electro statically actuated nanoplates. In the recent work done by Ansari et al. [38], the surface stress effects were also taken account of in vibrations analysis of circular nanoplates with various edge conditions. Concerning the applicability of the high-order continuum-based theories, Ghayesh et al. [39] used the strain gradient elasticity theory to study the nonlinear forced vibrations of the Euler–Bernoulli microscale beams. Also, the recent work of the same authors [40] was on the nonlinear dynamics of a microbeam in which the modified couple stress theory was employed to develop a size-dependent beam model.

To the Author's knowledge, no work on the bending and buckling of the nanoplates accounting for the effects of the surface properties and the shear deformation is available in the open literature. Herein, a size-dependent Mindlin plate model considering the surface effects is developed to investigate the buckling and bending behaviors of the nanoplates with various edge supports. The Gurtin–Murdoch surface theory and the first-order shear deformation theory are used to obtain the non-classical governing equations. By using the GDQ method, the governing equations are discretized and are then solved for different boundary conditions. Numerical results are presented to study the effects of the surface properties and the end conditions on the critical buckling load and the deflection of the nanoplate. From the present study, it is found that, depending on the selection of the surface properties, the stiffness of the nanoplate may decrease or increase due to the inclusion of the surface effects so that these changes become more significant at low thicknesses of the nanoplate.

2. Surface elasticity model for bending and buckling of the nanoplate

Consider a uniform nanoplate with the length a, width b and thickness h, as shown in Fig. 1. The parameters $\lambda = E\nu/(1-\nu^2)$ and $\mu = E/2(1+\nu)$ indicate the classical Lame constants, while the surface Lame constants are shown by λ^s and μ^s. Also, the parameters E, ν and τ^s denote Young's modulus, Poisson's ratio and surface residual stress, respectively. A coordinate system (x, y, z) is introduced at one corner of the mid-plane of the nanoplate so that the x axis is taken along the length of the nanoplate, the y axis in the width direction and the z axis is taken along the depth (thickness) direction. Also, the top and down surfaces of the nanoplate at $z = \pm h/2$ are denoted by S^+ and S^-, respectively. Because of the very small scale of the nanostructures, the non-classical continuum theories are not able to predict the mechanical response of the structures at nanoscale, accurately. Thus, modified continuum theories are required to accommodate the study of the structures at very small length scales. Herein, the Gurtin–Murdoch theory of elasticity is used to modify the first order sear deformation theory. From the FSDT, the general form of displacement components $(u_1,\ u_2,\ u_3)$ along the axes (x, y, z) is given by

$$u_1 = z\psi_x(x,y),\ u_2 = z\psi_y(x,y),\ u_3 = w(x,y) \tag{1}$$

where w is the transverse displacement, ψ_x and ψ_y represent the angular displacements in the x and y directions, respectively.

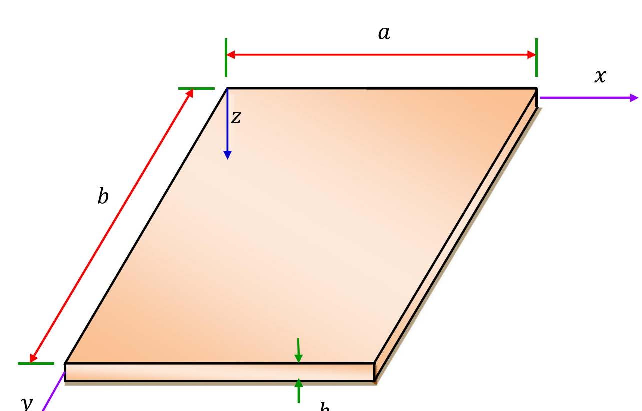

Fig. 1. Representation of a thin elastic plate used to model a nanoplate.