Physica E 57 (2014) 169–174

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Size-dependent dispersion characteristics in piezoelectric nanoplates with surface effects

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HIGHLIGHTS

- Surface piezoelectricity model is first introduced to investigate the dispersive properties of elastic waves propagating in piezoelectric nanoplates.
- The dispersive modes predicted by the theory of surface piezoelectricity exhibit obvious size-dependent behaviors.
- In the presence of surface effects, the magnitude of propagating velocity decrease considerably for a given frequency.
- The critical plate thickness was predicted, above which the surface effects may vanish.

ARTICLE INFO

Article history:
Received 17 August 2013
Received in revised form
15 October 2013
Accepted 10 November 2013
Available online 22 November 2013

Keywords:
Piezoelectric nanoplates
Surface effects
Dispersion characteristics
Size-dependence

ABSTRACT

In this paper, surface effects on the dispersion characteristics of elastic waves propagating in an infinite piezoelectric nanoplate are investigated by using the surface piezoelectricity model. Based on the surface piezoelectric constitutive theory, the presence of surface stresses and surface electric displacements exerting on the boundary conditions of the piezoelectric nanoplate is taken into account in the modified mechanical and electric equilibrium relations. The partial wave technique is employed to obtain the general solutions of governing equations, and the dispersion relations with surface effects are expressed in an explicit closed form. The impacts of surface piezoelectricity, residual surface stress and plate thickness on the propagation properties of elastic waves are analyzed in detail. Numerical results show that the dispersion behaviors in piezoelectric nanoplates are size-dependent, and there exists a critical plate thickness above which the surface effects may vanish.

1. Introduction

As a new emerging class of nanocomposites, piezoelectric materials at the nanoscale have been fabricated in various shapes and geometries for their technologically extensive applications in the nanoelectromechanical systems (NEMS), such as sensors/transducers, resonators, generators and piezoelectric field-effect transistors [1–4]. Different from the macroscpoic counterparts, the electroelastic properties of piezoelectric nanomaterials often exhibit distinct size-dependent behaviors due to the increasing ratio of surface area to volume, which has been demonstrated in the existing experimental observations [5,6] and atomistic simulations [7,8]. Therefore, it is important to accurately predict the static and dynamic behaviors of piezoelectric nanostructures with consideration of surface effects.

For elastic nanomaterials, the theory of surface elasticity established by Gurtin et al. [9,10] is widely adopted as a effective and reliable method to describe the surface effects on the size-dependent phenomena [11–14]. However, it should be mentioned that this theory will become invalid in estimating the electromechanical responses of piezoelectric nanostructures since it ignores the considerable sensitivity of piezoelectric effect near the surface area. As a pioneering wok, Huang and Yu [15] proposed a conceptual idea of surface piezoelectricity through extension of the surface elasticity. Subsequently, Pan et al. [16] developed a more precise surface theory to account for the linear superficial interplay between electricity and elasticity. In recent years, there has been significant interest in studying the electromechanical properties of piezoelectric nanostructures by using the surface piezoelectricity model [17–27]. Based on refined continuum mechanics theories, the bending behaviors of piezoelectric nanowires [17] as well as the vibration and buckling behaviors of piezoelectric nanobeams [18], nanofilms [19,20] and nanoplates [21,22] incorporating surface effects were analyzed under an prescribed electrical potential. Within the framework of nonlocal

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http://dx.doi.org/10.1016/j.physe.2013.11.007

electroelasticity theory, Wang and Wang [23] predicted the electromechanical coupling coefficient of the piezoelectric nanowire by using the piezoelectric surface-layer-based model. Chen [24] studied the overall thermoelectroelastic moduli of two-phase fibrous piezoelectric nanocomposites, and both the effects of surface stress and surface electric displacement were considered. In addition, Fang et al. discussed the surface effects on the dynamic strength around a piezoelectric nano-fiber [25] and a nano-particle [26], and on the effective dynamic properties of piezoelectric medium with randomly distributed piezoelectric nano-fibers [27].

Nowadays, two-dimensional piezoelectric nanoplates are regarded as suitable building blocks for most energy harvesting devices. Owing to the important applications of ultrasonic wave nondestructive testing in the detection on nanostructures, it is very urgent to investigate the mechanism of wave propagation in piezoelectric nanoplates. To the authors' best knowledge, the relevant literature is very limited except the paper of Zhang et al. [28], where only the case of anti-plane shear waves was considered. Motivated by this consideration, the present study aims to analyze the size-dependent properties of elastic waves propagation in an infinite piezoelectric nanoplate with consideration of surface effects. Numerical examples will show the influence of surface-related parameters on the dispersion curves with different plate thicknesses.

2. Basic equations of the surface piezoelectricity

At the nanoscale, the atomic structures in the vicinity of the piezoelectric surface are far from steady. Because of the broken symmetry, they automatically modulate to be a self-equilibrated state different from those in the underlying piezoelectric bulk. Accordingly, a piezoelectric surface is usually modeled as a two-dimensional heterogeneous membrane with its own material parameters, and the corresponding constitutive equations can be given as [17,29]

$$\sigma^s_{\alpha\beta}=\sigma^0_{\alpha\beta}+c^s_{\alpha\beta\gamma\delta}\varepsilon^s_{\gamma\delta}-e^s_{k\alpha\beta}E^s_k,\quad D^s_\alpha=D^0_\alpha+e^s_{\alpha\gamma\delta}\varepsilon^s_{\gamma\delta}+\kappa^s_{\alpha k}E^s_k, \tag{1}$$

where $\sigma^s_{\alpha\beta}$ and D^s_α are the surface stress and surface electric displacement, $\sigma^0_{\alpha\beta}$ and D^0_α are the residual surface stress and surface electric displacement without applied strain and electric field, $\varepsilon^s_{\gamma\delta}$ and E^s_k are the surface strain and surface electric field, and $c^s_{\alpha\beta\gamma\delta}$, $e^s_{k\alpha\beta}$ and $\kappa^s_{\alpha k}$ are the elasticity, piezoelectricity and permittivity tensor at the surface, respectively.

Assume that the piezoelectric surface layer of vanishing thickness adheres perfectly to the bulk without slipping. The equilibrium conditions on the surface are expressed according to the generalized Young–Laplace equations [30,31], as

$$\sigma^s_{\alpha\beta,\beta}=\sigma_{\alpha j}n_j-f_\alpha,\quad \sigma^s_{\alpha\beta}\zeta_{\alpha\beta}=\sigma_{ij}n_in_j-f_n, \tag{2}$$

$$D^s_{\alpha,\alpha}-D_jn_j=0, \tag{3}$$

where n_j and $\zeta_{\alpha\beta}$ denote, respectively, the outward normal vector and curvature of the surface, σ_{ij} and D_j are the stress and the electric displacement in the bulk, and $f=(f_1,f_2,f_n)$ stands for the prescribed traction on the surface. Throughout this paper, Einstein's summation convention is adopted for all repeated Latin indices (1–3) and Greek indices (1, 2).

3. Problem statement and formulation

Consider a piezoelectric nanoplate of hexagon crystal structure (class 6 mm) consisting of three layers: a bulk layer (middle) and two surface layers (upper and lower). A rectangular Cartesian coordinate system $ox_1x_2x_3$ as shown in Fig. 1 is used to describe the geometry of the problem. The nanoplate extends infinitely along x_1 and x_2 directions, and its upper and lower surfaces are defined by planes $x_3=h$ and $x_3=-h$, respectively. The piezoelectric body is poled along the x_3-axis perpendicular to the ox_1x_2 plane, and is subjected to a harmonic plane wave propagating in the x_2 direction.

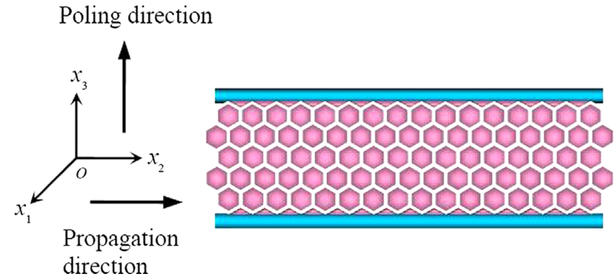

Fig. 1. Geometric configuration and coordinate systems of the problem.

In the absence of body forces and free charges, the equilibrium equations in the bulk can be written as

$$\sigma_{ij,j}=\rho\ddot{u}_i,\ D_{i,i}=0, \tag{4}$$

where ρ and u_i are the mass density and displacement of the bulk, respectively.

Following the classical piezoelectric theory [32], the constitutive relations in the bulk are

$$\sigma_{ij}=c_{ijkl}\varepsilon_{kl}-e_{kij}E_k,\quad D_i=e_{ikl}\varepsilon_{kl}+\kappa_{ik}E_k, \tag{5}$$

where c_{ijkl}, e_{kij} and κ_{ik} are the elastic, piezoelectric, and dielectric constants, respectively. The components of stain ε_{ij} and electric field E_i obey

$$\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}),\quad E_i=-\varphi_{,i}, \tag{6}$$

with φ being the electric potential. Eq. (6) is often referred to as the gradient equation of the piezoelectric materials, which is also valid for the surface layers.

The physical problem to be analyzed can be simplified into a two-dimensional model. For the elastic waves propagating in an infinite piezoelectric plate with transversely isotropy, SH wave (anti-plane strain state) may exist separately while P wave and SV wave (plane strain state) are coupled. Here, only a plane strain problem parallel to the x_2–x_3 plane is considered such that all field quantities are independent of x_1. Substitution of Eqs. (5) and (6) into Eq. (4) results in the following partial differential equations

$$\begin{aligned}&c_{11}u_{2,22}+c_{44}u_{2,33}+(c_{13}+c_{44})u_{3,23}+(e_{31}+e_{15})\varphi_{,23}=\rho\ddot{u}_2,\\&(c_{44}+c_{13})u_{2,23}+c_{44}u_{3,22}+c_{33}u_{3,33}+e_{15}\varphi_{,22}+e_{33}\varphi_{,33}=\rho\ddot{u}_3,\\&(e_{15}+e_{31})u_{2,23}+e_{15}u_{3,22}+e_{33}u_{3,33}-\kappa_{11}\varphi_{,22}-\kappa_{33}\varphi_{,33}=0.\end{aligned} \tag{7}$$

According to the partial wave technique, the general solutions with respect to the displacement u_i and electric potential φ in Eq. (7) can be expressed in the form [33]

$$\begin{aligned}&u_2=A\exp(kbx_3)\cos k(x_2-ct),\\&u_3=B\exp(kbx_3)\sin k(x_2-ct),\\&\varphi=C\exp(kbx_3)\sin k(x_2-ct),\end{aligned} \tag{8}$$

where A, B, and C are constant coefficients, b is a parameter to be determined, k is the wave number, c is the phase velocity, and $\omega=kc$ stands for the circular frequency of the elastic waves.