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Accurate tool for express optical efficiency analysis of cylindrical light-tubes with arbitrary aspect ratios

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ABSTRACT

The vertical straight pipes are a desirable technology for delivery daylight into interior spaces because this concept benefits from luminance distribution of the whole upper hemisphere, including sunlight that is most important source of light under clear sky conditions. Vertical pipes can transport sunbeams to deep interiors independent of solar azimuth and/or room orientation. Currently, the tools for modelling different pipes are widely available, but their accuracy decreases as the aspect ratio of a light-pipe increases. In sunny environments with high solar altitudes and favorable clear skies the tubular light-guides allow to deliver daylight into deep offices, halls, or even underground spaces in building cores. The optical efficiency of such systems strongly depends on number of reflections the beams undergo on their path from the top to the bottom of a light pipe. In general, dense-grid and high CPU requirements meet when accurate numerical predictions are required. Such computations are non-attractive if intended for routine (mass) modelling.

In this paper we introduce an analytical solution to the optical efficiency of straight pipes that is applicable to all aspect ratios and provides accurate predictions with very low demands on processor time. The new analytical model is validated and benchmarked against accurate HOLIGILM calculations, while showing the percent deviations are kept below 10% for most cases studied.

1. Introduction

A couple of parametric studies indicate problems with daylight availability in complex buildings situated in some geographical locations (Reinhart and Wienold, 2011; Dubois and Flodberg, 2013; Cammarano et al., 2014). Daylight is largely missing in underground spaces, building cores or deeper parts of interiors. Hollow circular light pipes represent a common concept for delivery daylight into interior spaces (Jenkins and Muneer, 2003; Aizenberg, 2009; Kocifaj et al., 2012; Samuhatananon et al., 2011). Among various installations (Edmonds, 2010; Taengchum et al., 2014), the tubular vertically directed pipe is the most traditional design that still deserves a special attention and further investigations because this is the only possibility how to collect the light over the whole sky hemisphere (Al Marwaee and Carter, 2006; Mohelníková, 2009; Malet-Damour et al., 2014; Carter, 2014). A pipe bend can result in light losses (van Derlofske and Hough, 2004) or even in blind angles at which the light beams are unable to transit from the upper to the bottom interface (Kocifaj et al., 2010). In contrast, a bend can also increase the efficiency of a pipe if directed towards the sun, but this is only a singular momentary state. The optimum conditions cannot be guaranteed for a whole day because the light-guide is firmly embedded in the building roof, but the sun traverses the sky on different paths depending on season and geographical position of a site. A zenith-directed straight pipe is a trade-off between different duct configurations as it benefits from sunlight for any sundisk position.

The energy losses are typically small if ratio of length to diameter (so-called aspect ratio) is generally low for a straight pipe. This is because the light beams undergo only a few or even no one reflection event on their path from the cupola to the light pipe base. This is why most of empirical tools are quite successful in predicting the light pipe efficiency (Jenkins et al., 2005). However, the luminous flux at the bottom interface of a light pipe decreases rapidly as the aspect ratio approaches large values and this trend does not change significantly even if a tube is manufactured with internally high reflective surfaces. Light transmission through long tubes is difficult to predict using simple empirical tools and this is a reason for why these predictions diverge from each other and only a few of them (accidentally) match the trend which arise from exact computations. A use of exact numerical tools might not be advantageous under some circumstances, e.g. in cases

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when express analysis of optical efficiency is needed. The light is sometimes transported deeper into building – 10 m or even deeper below a ceiling level (Mayhoub, 2014; Garcia-Hansen and Edmonds, 2015). Usefulness of such a pipe in practical realizations strongly depends on many factors, specifically internal coating and its reflectance, surface imperfections, deflection from mirror-type reflectivity, etc.

Empirical tools are scarcely appropriate to model optical efficiency of long pipes, thus exact solution methods are often the only way to predict the luminous flux at the bottom interface of a light tube. In this paper we introduce the first time the analytical solution to the problem that is applicable to both, cloudy and cloudless sky conditions. The latter is of high importance because the peak illuminance levels are typically achieved in sunny days. The model developed here is based on the standardized CIE skies in which a twin set of gradation functions and indicatrix functions describes sky luminance distributions (Kittler, 1999).

2. Theoretical derivations

The optical efficiency η of a light-tube is computed as a ratio of the luminous flux F_2 passing through the bottom of the tube to the luminous flux F_1 entering the top of the tube, i.e.

$$\eta = \frac{F_2}{F_1} \tag{1}$$

A transparent cupola mounted at the upper interface influences F_1 and F_2 in the same way, thus having no effect on η.

Consider the height of the vertical cylindrical tube is H, while the tube radius is R. Let r_0 and φ_0 are polar coordinates with $r_0 = 0$ being the center of the coordinate system (i.e. the point in which light-pipe axis crosses circular opening of the tube) and $r_0 = R$ is applicable to all points at the edges of the circular cross-section. The luminous flux $F_i (i = 1,2)$ through the upper and/or the lower aperture of the tube is then

$$F_i = \int_0^R \int_0^{2\pi} E_i(r_0,\phi_0) r_0 d\phi_0 dr_0 \tag{2}$$

where $E_i(r_0,\phi_0)$ is the illuminance at (r_0,ϕ_0). For the upper aperture we have

$$E_1(r_0,\phi_0) = \int_0^{2\pi} \int_0^{\pi/2} L(\vartheta,\alpha)\cos\vartheta\sin\vartheta d\vartheta d\alpha \tag{3}$$

where $L(\vartheta,\alpha)$ is the sky luminance at zenith distance ϑ and azimuth angle α. Analogously, the illuminance at the light tube base is

$$E_2(r_0,\phi_0) = \int_0^{2\pi} \int_0^{\pi/2} J(\vartheta,\phi,r_0,\phi_0)\cos\vartheta\sin\vartheta d\vartheta d\phi \tag{4}$$

The luminance J from the tube as seen in direction (ϑ,ϕ) is

$$J(\vartheta,\phi,r_0,\phi_0) = L\rho^{N(r_0,\phi_0,\vartheta,\phi)} \tag{5}$$

where L is the luminance of a sky element, ρ is the reflectance of the internal surface of light pipe, and N is the number of reflections. The geometry relations and other information on the problem solved can be found in (Kocifaj et al., 2008).

The number of reflections N (see Kocifaj et al., 2008) can be, after some algebra, expressed as a function of (r_0,ϑ,ϕ). When we omit the Int operation and apply adequate Taylor series expansion to some particular functions by the parameter $(r_0/R)\sin\phi$, we get the following real-valued approximation

$$N(r_0,\vartheta,\phi) \cong \frac{1+\xi}{2} - \frac{\beta}{2}\cos\phi + \frac{1}{4}\xi\beta^2\sin^2\phi + \frac{3}{16}\xi\sum_{k=2}^{6}\beta^{2k}\sin^{2k}\phi$$

$$\beta = \frac{r_0}{R}$$

$$\xi = \frac{H}{R}\tan\vartheta \tag{6}$$

where H/R is double of what we know as aspect ratio (see e.g. Carter, 2002).

The luminous energy entering the tube is due to a non-trivial superposition of diffuse light of sky and direct sunlight. The latter dominates all light beams under clear sky conditions except for extremely low sun positions. Therefore we analyze the contribution of direct sunbeams and diffuse light of sky separately.

2.1. Direct sunlight

The zenith angle of a sunbeam is conserved on its path from the top to the bottom interface of a vertically oriented straight pipe. However, the azimuth angle of a light beam changes with each reflection event, so the beams escaping the tube at its base typically have no preferred direction if N is large. The directionality of light can, however, be an important factor for short pipes, while having no effect on the luminous flux. This is because of symmetry relations – the direction of light beams below a pipe would change with the azimuth of sun, but the luminous energy crossing the light-tube base would stay constant. We take advantage of this model and replace the sundisk by a uniformly bright solar almucantar that produces the horizontal luminous flux equal to that produced by the sun. This tricky approach allows for significant theoretical simplifications. The hypothetical luminance of the solar almucantar is then simulated by a formula

$$L_S(\vartheta) = \frac{P_v}{\cos\vartheta_S}\frac{\delta(\vartheta-\vartheta_S)}{2\pi\sin\vartheta} \tag{7}$$

where ϑ_S is solar zenith angle, P_v is the direct solar illuminance on horizontal plane and $\delta(.)$ is the Dirac delta function. It can be easily proven that Eq. (7) and the formula introduced by Kocifaj et al. (2008) for sundisk yield the same horizontal illuminance P_v and the same fluxes at the light tube base. No doubt that the luminous flux at the upper interface of a light tube is

$$F_{S,1} = \pi R^2 P_v \tag{8}$$

Using a simple algebra and after a bit of manipulations using Eqs. (2), (4)-(7) we obtain

$$F_{S,2} = \pi R^2 P_v \rho^{\frac{1+\xi_S}{2}}(1 + p\xi_S\ln\rho + q\xi_S^2\ln^2\rho)$$

$$\xi_S = \frac{H}{R}\tan\vartheta_S$$

$$p = 0.124573$$

$$q = 0.025514 \tag{9}$$

where the values of p and q are results of numerical integration (not shown here).

The optical efficiency of a straight vertical pipe for direct sunbeams is then

$$\eta_S = \rho^{\frac{1+\xi_S}{2}}(1 + p\xi_S\ln\rho + q\xi_S^2\ln^2\rho) \tag{10}$$

Most importantly, Eq. (10) implies the light pipe efficiency is only ruled by two parameters, specifically ξ_S and ρ. We expect the formula (10) should work well if $\xi_S > 2$ and its accuracy will further improve with increasing the length H for given R.

2.2. Diffuse light

The diffuse component of ground-reaching light has the most uncertain contribution to the total light field. This is because the angular distribution of scattered light strongly depends on many factors, such as turbidity of atmospheric environment, microphysical properties of aerosol particles, cloud distribution, and humidity. It is extremely difficult to model luminance distribution in such heterogeneous environment. A set of simplifications has been introduced of which the most