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Steady flow past a torus with aspect ratio less than 5

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Steady flow past a torus with an aspect ratio less than 5 and its axis aligned with the flow is studied numerically by solving the steady, axisymmetric Navier–Stokes equations. The wake structure behind tori exhibits diverse behaviours. The *detached recirculating zone on the axis*, the *attached recirculating zone*, and the *detached recirculating zone behind the torus tube* may appear individually or concurrently, depending on the aspect ratio and the Reynolds number. A wake structure map is summarized based on the observed flow behaviours. Six flow regimes with different wake behaviours are identified and the corresponding flow regime map is plotted, which include the *no-recirculating-zone regime*, the *single-detached-recirculating-zone regime*, the *single-attached-recirculating-zone regime*, the *two-recirculating-zone regime I*, the *two-recirculating-zone regime II*, and the *three-recirculating-zone regime*. Over the range of aspect ratio $1.9 < AR < 2.4$, the detached wake initially increases but then decreases in size with Reynolds number, and eventually disappears at Reynolds numbers beyond a critical value (depending on the aspect ratio). The underlying mechanisms of the onset and disappearance of the recirculating zones are discussed in terms of vorticity accumulation and base bleed. The recirculating zone first occurs when the maximum vorticity on the surface of the torus exceeds about 5. The *detached recirculating zone on the axis* of the torus disappears once the flow rate through the hole of the torus is beyond a certain threshold. In addition, the present results suggest that different transition modes to non-axisymmetric flow for tori with different aspect ratios reported in the literature may result from the wake structures prior to the transition.

1. Introduction

The flow past bluff bodies, such as cylinders with various cross-section shapes (e.g. Leweke et al., 1993; Ganga Prasath et al., 2014; Rastgou and Saedodin, 2013; Williamson, 1996), have been extensively studied for a long time, with a wide range of analytical, experimental and numerical methods. The present study concerns the steady flow around a torus. The geometry of the torus (Fig. 1) can be characterised by its aspect ratio $AR = D/d$, where d is the cross-sectional diameter of the torus and D is the centre-line diameter of the torus. Note that the torus becomes a sphere at $AR = 0$ while a circular cylinder at the limit $AR \to \infty$ (Sheard et al., 2003). Thus, from an academic point of view, the studies of flow past tori at various aspect ratios can provide some insight into the geometric effect on bluff-body flow.

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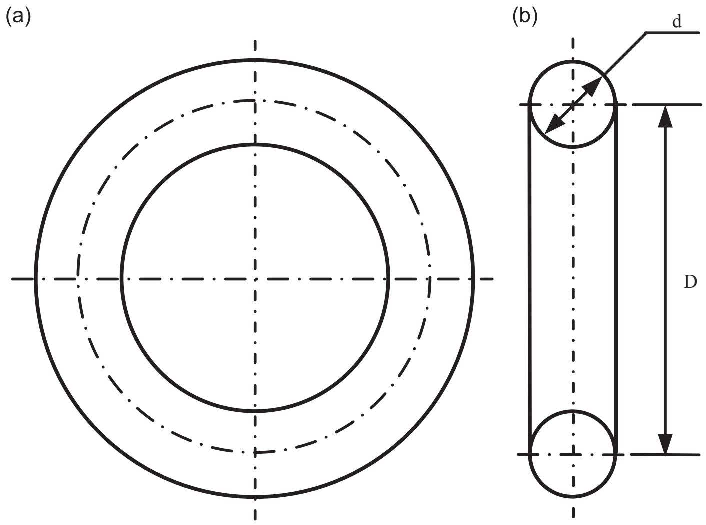

Fig. 1. Schematic representation of a torus; (a) top view; (b) side view.

It is known that the wake behind a cylinder is affected by 'end effects' resulting from the finite cylinder length in experiment, even for very long cylinders (Leweke and Provansal, 1995; Williamson, 1996). A torus (a body with a spanwise periodicity but without ends) can avoid such 'end effects'. Roshko (1954) conducted one of the earliest studies on the flow past a torus. His measurements indicated that the vortices shed from a torus with $AR = 10$ are almost the same as those shed from a straight circular cylinder. However, for a torus with $AR = 5$, the vortex shedding behaves somewhat differently. Bearman and Takamoto (1988) confirmed the conclusion of Roshko (1954) by investigating the wake behaviour behind tori of non-circular cross-section. Monson (1983) pointed out that there may be more than one wake configuration when investigating the drag on a torus freely falling in a viscous fluid. Leweke et al. (1993) and Leweke and Provansal (1994, 1995) then conducted a series of systematic studies to explore possible wake configurations behind tori with a high aspect ratio at a low Reynolds number using both experiments and application of the Ginzburg–Landau model. More recently, Sheard et al. (2003, 2004a, 2004b) performed detailed studies on flow past tori by both axisymmetric and non-axisymmetric simulations as well as linear Floquet stability analysis. The flow and transition characteristics over the whole aspect ratio range $0 \le AR \le \infty$ were summarized. The development of a subharmonic three-dimensional instability in a vortex street downstream of a torus was investigated both numerically and experimentally by Sheard et al. (2005b). They confirmed that the subharmonic instability does not initiate a period-doubling cascade in the wake by performing simulations at higher Reynolds numbers. Sheard et al. (2005a) also provided detailed data on variations in the drag coefficient for low-Reynolds-number flow past tori.

Leweke and Provansal (1995) found that the wake behind tori with high aspect ratios remains stationary for Reynolds numbers less than approximately 50. This was confirmed by the stability analysis and numerical simulations of Sheard et al. (2003, 2004a), which showed that, for tori with $AR > 1$, the transition Reynolds number decreases with aspect ratio and asymptotically approaches that of a circular cylinder for $AR \to \infty$. Sheard et al. (2003) also found an interesting physical phenomenon, a detached recirculating bubble on the axis downstream of the torus, occurring over aspect ratios $1 < AR \le 2$.

The aforementioned studies have shown that the wake behaviour varies with the aspect ratio and the Reynolds number. However, the position and orientation of the steady recirculating zone are difficult to measure consistently (Sheard et al., 2003). Thus, one of the motivations of the present work is to present a systematic study on the wake structure behind a torus. Tori with aspect ratios of $1 < AR \le 5$ are examined as the wake structure behind tori with higher aspect ratios have been reported to be rather similar to that of a circular cylinder.

Another issue of interest is the detached recirculating zone on the axis downstream of the torus. The similar detached recirculating zones have also been reported in other flow conditions. The first is the flow around a two-dimensional cylindrical body with "base bleed", which has been investigated by Leal and Acrivos (1969). It was found that the detached recirculating zone appears at a Reynolds number of ~ 260. The second example is the translational flow motion of a viscous drop (Dandy and Leal, 1989), in which the detached recirculating zone occurs across a wide range of parameters. Other examples include the wake downstream of the porous bluff bodies, e.g. square cylinder (Yu et al., 2010), circular cylinder (Yu et al., 2011), as well as the porous sphere (Yu et al., 2012). Leal (1989) and Dandy and Leal (1989) suggested that the appearance of the recirculating zones is not a result of separation in boundary layers in an adverse gradient but may be due to vorticity accumulation. Thus, another motivation of the present study is to shed some insight on this aspect.

In the remainder of this paper we will numerically simulate the steady axisymmetric flow past a torus with $AR < 5$. The organization of this paper is as follows: Section 2 provides a succinct description of the governing equations, boundary conditions, and numerical techniques. Section 2 also presents a grid independence study and code validation. This is followed by a presentation and discussion of the numerical results in Section 3, which include variations in flow patterns