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Loads and propulsive efficiency of a flexible airfoil performing sinusoidal deformations

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ABSTRACT

This paper presents the application of state-space airloads theory to a flexible airfoil performing sinusoidal deformations at high Reynolds numbers. Given the two-dimensional motion of a flexible airfoil, we derived the closed forms for the propulsive force, lift force, generalized forces of pitching and bending as functions of reduced frequency k, dimensionless wavelength z, and dimensionless amplitude $A/(2b)$. We also calculate the power required to perform this sinusoidal deformation and the propulsive efficiency. Our results show a positive, time-averaged propulsive force for all $k > k_0 = \pi/z$, which is when the wave speed is greater than the moving speed. At $k = k_0$, which is when the moving speed reaches the wave speed, the motion reaches a steady-state with all forces being zero. When $k < k_0$, the system is the case of energy extraction in which the drag force (negative propulsive force) and wake are causing the airfoil to vibrate. For the propulsive case, the propulsive efficiency decreases from 1.0 to 0.5 as k goes to ∞, or k_0 goes to 0. If there were no wake, the propulsive force would be zero at wavelengths of $z=0.569$ and $z=1.3$ for all k, and local optimum at $z=0.82$. Though these extrema of propulsive force with wavelength are smoothed out by the wake effect, one can still see around $z=1.3$ ($k=2.4$) the slope transitions of all three powers in Fig. 9. When $k < 2.4$, the cost for high propulsion become more expensive as more power input is used by wake, thus less efficiency.

1. Introduction

In the Ph.D. work of the first author (Ulrich, 2012), experimental measurements give numerical values for typical body undulations of *Caenorhabditis elegans* swimming in viscoelastic fluids. Other literature on the locomotion of similar spermatozoa and nematodes (at low Reynolds numbers) is quite extensive (Hancock, 1953; Gray and Hancock, 1955; Gray and Lissmann, 1964; Karbowski et al., 2006; Korta et al., 2007). It is well-known that such creatures produce a sinusoidal wave-like motion for locomotion at low Reynolds numbers. However, it is also observed that creatures operating at large Reynolds numbers (such as sea snakes and fish) display a similar undulatory motion (Shine and Shetty, 2001; Maladen et al., 2011). Therefore, it would seem appropriate to study the effect of sinusoidal undulations on locomotion at large Reynolds numbers (i.e., in potential flow).

Indeed, the application of potential flow theory to fish swimming does have a rich history. Investigations go back to the work of Lighthill (1960). The classic work on the locomotion due to thin-body, small deformations in potential flow is

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Nomenclature

A	amplitude, m
b	semi-chord, m
$C(k)$	Theodorsen Function
C_F	coefficient of propulsive force or propulsive power, $C_F = -D/(2\pi\rho b u_0^2)$
$C_{F-\text{no wake}}$	coefficient of propulsive force without wake effect
$C_{F-\text{with wake}}$	coefficient of propulsive force with wake effect
C_L	coefficient of lift force, $C_L = L_0/(2\pi\rho b u_0^2)$
C_{L1}	coefficient of generalized force of pitching, $C_{L1} = L_1/(2\pi\rho b u_0^2)$
C_{L2}	coefficient of generalized force of bending, $C_{L2} = L2/(2\pi\rho b u_0^2)$
C_P	coefficient of power required for airfoil deformations, $C_P = P/(2\pi\rho b u_0^3)$
C_W	coefficient of power lost due to wake
ΔC_F	coefficient of propulsive force introduced by wake alone
D	drag per unit span, N/m
f	frequency, Hz
h	equation of deformation of a flexible airfoil, down is positive, m
h_n	components of h in Chebyshev polynomials, $n = 0, 1, 2, \ldots, \infty$, m
i	the imaginary unit
$Im[]$	the imaginary part of a complex number
k	reduced frequency, $k = \omega b/u_0$
k_0	critical reduced frequency, $k_0 = \pi/z$
L_0	lift per unit span, N/m
L_1	generalized force of pitching per unit span, N/m
L_2	generalized force of bending per unit span, N/m
M	the real part of $1-C(k)$
N	the imaginary part of $1-C(k)$
P	power exerted per unit span to move the wing, N/s
Re[ ]	the real part of a complex number
t	time, s
u_0	x-velocity of flow relative to the reference frame, m/s
w_n	components of total velocity field, m/s
(x, y)	Cartesian coordinates of the reference frame, m
X	normalized reduced frequency, $X = k/(k+1)$
z	the normalized wavelength, $z = \Lambda/(2b)$
α_n	cosine factor of h_n/b
β_n	sine factor of h_n/b
ε	power efficiency
Λ	wavelength, m
λ_0	the velocity due to shed vorticity, m/s
λ_α	cosine factor of λ_0/u_0
λ_β	sine factor of λ_0/u_0
ρ	density of surrounding fluid or air, kg/m^3
τ	reduced time, $\tau = u_0 t/b$
φ	Glauert variable, rad
ω	angular frequency, $\omega = 2\pi f$, rad/s
$\dot{()}$	$\partial()/\partial t$
$\ddot{()}$	$\partial^2()/\partial t^2$
$()'$	$\partial()/\partial\tau$
$()''$	$\partial^2()/\partial\tau^2$

attributed to Wu (1971a,b). Later, others extended the work to large motions with further applications to the swimming of fish (Melli-Huber et al., 2004; Melli, 2008; Kanso, 2009). All of these studies involve numerical solution of the potential-flow equations. In fact, the initial work on small-motion locomotion (Wu, 1971a) has no figures or numerical results, and the second work (Wu, 1971b) has only a few numerical results for optimum motions. Therefore, additional insight might be gleaned if one could find closed-form solutions for the locomotion of a slender body in potential flow.

Recent developments in the theory of deformable airfoils offer the promise of developing such closed-form expressions for the propulsive force, generalized deformation forces, and propulsive efficiency of a deforming airfoil in potential flow. In particular, it has been shown that the generalized Theodorsen function (applicable to a rigid airfoil) can be written in terms of simple state-variable equations (Peters et al., 1995; Peters, 2008). It has also been shown that this approach can be extended to a completely deformable airfoil (Peters et al., 2007). The general Theodorsen function is, indeed, the same as found in earlier work (Wu, 1971a) except that it is from state equations. The theory is particularly attractive because it is in a closed-form matrix format that allows easy assembly with structural dynamic equations for a complete fluid/structure interaction between the airfoil and the flow.

In the case of airfoil deformations limited to plunge, pitch, and trailing or leading-edge flaps, the theory has been shown to reduce to classic theories for pitch and moment (Wagner, 1925; Theodorsen, 1934), for drag and propulsive force (Garrick, 1936), for unsteady free-stream (Issacs, 1945, 1946; Greenberg, 1947), and for rotary-wing loads (Loewy, 1957). The theory has also been validated against experimental data and CFD results for airfoils with trailing-edge flaps in dynamic stall (Ahaus and Peters, 2010; Ahaus et al., 2010). Thus, this theory has now found its way into the codes of Sikorsky, Bell Helicopter, Advanced Rotorcraft Technology (Flightlab), and the U. S. Army (RCAS). However, the complete general form of the theory for complete arbitrary airfoil morphing has not heretofore been exercised. Thus, application of the theory to sinusoidal locomotion promises new and interesting insights into the locomotion problem.

The purpose of this paper is to show how this new, general formulation for deforming airfoils can yield closed-form solutions for the propulsive force, generalized loads, and propulsive efficiency of airfoils undergoing sinusoidal locomotion. These closed-form results are possible because the theory is developed rigorously from the potential flow equations by use of a Glauert expansion (von Kármán and Burgers, 1935). The Glauert expansion is based on closed-form solutions to Laplaces