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Time dependent flexural gravity waves in the presence of current

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A R T I C L E I N F O

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A B S T R A C T

In the present study, a combined effect of current and compressive force on time dependent flexural gravity wave motion in both the cases of single and two-layer fluids is analyzed in finite and infinite water depths in two dimensions. The roots of the dispersion relation associated with the plane flexural gravity waves are analyzed via contour plots and by plotting various terms of the dispersion relation separately. The characteristic of plane flexural gravity waves is studied by analyzing the phase and group velocities along with the law of conservation of energy flux to understand the combined effect of current and compressive force on the wave motion. The integral form of the time dependent Green's function in the presence of current is obtained using the Laplace transform method and used in Green's identity to derive the time dependent velocity potential for the flexural gravity wavemaker problem. The time harmonic Green's function and velocity potentials are obtained as a special case from the time dependent problems. Numerical results are computed and analyzed in particular cases using the method of stationary phase to obtain the asymptotic results for Green's function and the deflection of ice sheet. The integral form of Green's function derived here will be suitable to deal with physical problem when the roots of the dispersion relation for the flexural gravity wave problem coalesces which were otherwise not possible in the eigenfunction expansion method used for time harmonic problems.

1. Introduction

It is observed that throughout the world near the coastal regions, currents of speed exceeding 1 m/s move along with the waves that are generated due to the natural phenomena such as change of wind speed and direction, tidal, thermal and coriolis effects. Various aspects of wave motion in the presence of uniform current have been analyzed by Taylor (1955). Peregrine (1976) highlighted different mathematical approaches, with relevant observations for varied physical circumstances associated with wave current interaction. Wave propagation in the presence of current over a varying sea bed has been studied by Jonsson and Wang (1980), and Craik (1985). Hartnack (2000) analyzed various aspects of small amplitude water wave motion in the presence of non-uniform current speed using conservation of mass, momentum and energy. All these studies are related to gravity wave interaction with uniform/non-uniform current in homogeneous fluid medium of constant density. On the other hand, Alam et al. (2009) derived the velocity potential for the density-stratified fluid associated with the moving source point in the upper layer as well as in the lower layer. Nguyen and

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Yeung (2011) derived the time dependent velocity potentials assuming an unsteady point source on the upper layer fluid.

However, in recent decades, there has been considerable interest in analyzing the interaction of gravity waves with very large floating structures due to their wide applications in ocean engineering and cold region science and technology. Surface gravity waves propagating in the open ocean penetrate into the region covered by sea ice or the region covered by flexible structures to generate flexural gravity waves. A significant progress on wave structure interaction problems has been made under the assumption that the motion is simple harmonic in nature (as in Manam et al., 2006; Mondal et al., 2013). Manam and Kaligatla (2012) studied the effect of varying bottom on the membrane coupled gravity waves using mild-slope approximation. The hydroelastic analysis of gravity wave interaction with concentric porous and flexible cylinders is studied by Mandal et al. (2013). An effect of compression on flexural gravity waves is analyzed by Schulkes et al. (1987) and recently Mohapatra et al. (2013) studied the effect of compression on diffraction of surface waves by floating elastic plate. Apart from wave structure interaction problems in single homogeneous fluid, efforts have been made to study the wave structure interaction problems in two-layer fluid having a plate covered surface and an interface. Bhattacharjee and Sahoo (2008a) derived the expansion formulae for wave structure interaction problems in two-layer fluid in two dimensions and demonstrated the utility of the expansion formulae by analyzing the wave scattering by cracks in a floating ice sheet. Recently, Mondal and Sahoo (2012) generalized the expansion formulae to deal with wave structure interaction problems in three dimensions and analyzed various characteristics of the eigen-systems. These studies are based on the assumption that the motion is simple harmonic in nature. In the case of two-layer fluid, apart from the plate covered free surface there is an interface which gives rise to the propagation of flexural gravity waves in the surface and interface modes leading to the existence of two plane progressive waves.

On the other hand, Schulkes and Sneyd (1988) studied the time dependent response of floating ice sheet when a point load is moving steadily on the floating ice sheet. Squire et al. (1996) demonstrated various aspects of time dependent wave motion on moving loads on flexural gravity waves. Effects of uniform current on plane flexural gravity waves are studied by Bhattacharjee and Sahoo (2007). Further, Bhattacharjee and Sahoo (2008b) analyzed the transient time dependent flexural gravity wave motion in the presence of current using the combined Fourier–Laplace transforms and the method of stationary phase. Meylan (2002) derived the spectral theory of a thin plate floating on shallow water and used to solve time dependent wave motion. Meylan and Sturova (2009) presented three solution methods to deal with a class of time dependent initial value problems associated with wave motion in the presence of a floating elastic plate in two dimensions. In Chapter 5, Sahoo (2012) briefly demonstrated various methods to deal with time dependent wave structure interaction problems. Hazard and Meylan (2008) derived the spectral theory for a thin plate floating on finite water and used to solve time dependent wave motion.

Another interesting class of problems related to time dependent flexural gravity wave motion is the transient flexural gravity wavemaker problems. Bukatov (1980) studied the combined effect of current and compressive force on the generation of time dependent transient flexural gravity waves due to the presence of time harmonic normal stresses acting on the free surface using the Fourier transform method in a homogeneous fluid of constant density in finite water depth. It was observed that the basic characteristics of flexural gravity waves, generated by time harmonic pressure moving over the ice plate, depend on the velocity of current, frequency of oscillation of the stress acting on the plate surface and the magnitude of the compressive force. Lu and Dai (2008) investigated the dynamic characteristics of time dependent flexural and capillary gravity waves due to fundamental singularities in water of finite depth. Using the combined Laplace–Fourier transform, the velocity potentials and surface elevations are obtained in integral forms from which the asymptotic results for large time and space are obtained using the method of stationary phase. Montiel et al. (2012) analyzed the transient response of floating elastic plates to wavemaker forcing in two dimensions using the Fourier transform technique in both time and frequency-domain.

In the present paper, a combined effect of current and compressive force on the gravity wave interaction with floating flexible structure is analyzed in time-domain in both the cases of single-layer and two-layer fluid under the assumption of small amplitude water wave theory and structural responses. The problems are analyzed in both the cases of finite and infinite water depths. Combined effects of current speed and compression on plane flexural gravity wave motion are studied by analyzing the dispersion relation, phase and group velocities. Further, location of various roots is demonstrated in specific physical problems using contour plots which also act as initial guess for finding the roots of the complex dispersion relations.

Using the Laplace transform method, expansion formulae for the transient Green's function for the wave structure interaction problem in the presence of current and compressive force are obtained. Applying Green's identity to the velocity potential and Green's function, integral representation of the velocity potentials is obtained for the transient flexural gravity wavemaker problem. From the integral form the transient Green's function and velocity potential, time-harmonic solutions are derived in particular cases using the characteristics of the roots of dispersion relations. Asymptotic results are obtained in special cases using the method of stationary phase. Various numerical results are presented to understand the combined effect of current and compressive force on the flexural gravity wave motion. The integral form of the time dependent Green's function and the velocity potential for the transient flexural gravity wavemaker problem is likely to play a significant role in converting complex problems of realistic nature into integral and/or integro-differential equations whose solution can be obtained numerically and a similar approach can be used to deal with acoustic wave interaction with flexible structures.