Journal of Fluids and Structures 45 (2014) 124–137

Contents lists available at ScienceDirect

Journal of Fluids and Structures

journal homepage: www.elsevier.com/locate/jfs

Bifurcation and chaos of a flag in an inviscid flow

Ming Chen [a], Lai-Bing Jia [a], Yan-Feng Wu [a], Xie-Zhen Yin [a,*], Yan-Bao Ma [b]

[a] Department of Modern Mechanics, University of Science and Technology of China, Hefei, Anhui 230027, PR China
[b] University of California, Merced, CA 95343, USA

ARTICLE INFO

Article history:
Received 4 October 2012
Accepted 25 November 2013
Available online 31 December 2013

Keywords:
Fluid–structure interactions
Flutter instability
Bifurcation
Chaos

ABSTRACT

A two-dimensional model is developed to study the flutter instability of a flag immersed in an inviscid flow. Two dimensionless parameters governing the system are the structure-to-fluid mass ratio M^* and the dimensionless incoming flow velocity U^*. A transition from a static steady state to a chaotic state is investigated at a fixed $M^*=1$ with increasing U^*. Five single-frequency periodic flapping states are identified along the route, including four symmetrical oscillation states and one asymmetrical oscillation state. For the symmetrical states, the oscillation frequency increases with the increase of U^*, and the drag force on the flag changes linearly with the Strouhal number. Chaotic states are observed when U^* is relatively large. Three chaotic windows are observed along the route. In addition, the system transitions from one periodic state to another through either period-doubling bifurcations or quasi-periodic bifurcations, and it transitions from a periodic state to a chaotic state through quasi-periodic bifurcations.

1. Introduction

Complex interactions of flexible plates with ambient fluid are common in nature, e.g., a flag flapping in the wind and a tree leaf falling in the air. In industry, the flutter of paper has been widely observed in printing processes (Watanabe et al., 2002b). In biomedicine, snoring is the result of flow-induced vibration of the soft palate (Aurégan and Depollier, 1995; Huang, 1995).

Due to its practical and theoretical significance, the plate–fluid interaction problem has been studied extensively both experimentally and theoretically. Experimental observations of flag flutter have been conducted in soap films (Zhang et al., 2000; Jia et al., 2007; Ristroph and Zhang, 2008), water tunnels (Shelley et al., 2005) and low-speed wind tunnels (Taneda, 1968; Datta and Gottenberg, 1975). By placing a filament in a flowing soap film, Zhang et al. (2000) studied the dynamics of a filament immersed in two-dimensional flow. Two distinct states were found: one is a static stable state for a short filament, and the other is periodic flapping state for a sufficiently long filament. Watanabe et al. (2002b) experimentally investigated the dynamics of a sheet in a low-speed wind tunnel. The results showed that the two key factors determining the flutter are the dimensionless bending stiffness of the sheet and the mass ratio between the fluid and the structure. Ait Abderrahmane et al. (2012) experimentally studied the flapping dynamics of a flag in relatively high turbulent flow. Periodic, quasi-periodic and chaotic vibrations were observed in the study. When Shelley et al. (2005) studied the dynamics of a heavy flag interacting with flowing water, it was found that the compliant sheet begins to flap with a Strouhal frequency consistent with animal locomotion. The subcritical Hopf bifurcation was reported separately in these experimental studies. Eloy et al. (2012) showed experimentally that inevitable planeity defects of the flags lead to large hysteresis in the experiments. The influence of the flag shape was experimentally and theoretically studied (Eloy et al., 2007, 2008), and the

* Corresponding author.
E-mail address: xzyin@ustc.edu.cn (X.-Z. Yin).

0889-9746/$ - see front matter
http://dx.doi.org/10.1016/j.jfluidstructs.2013.11.020

results showed that the two-dimensional limit cannot be achieved experimentally because of gravity and 3D effects. The authors also addressed the importance of the flag aspect ratio while predicting the instability threshold.

Different methods have been proposed to numerically simulate the flag-like problem. The most widely used method is the Immersed Boundary (IB) method. Using this method, Zhu and Peskin (2002) studied the dynamics of a filament immersed in a flowing soap film. In the study, it was found that the increase of the mass or length of the filament destabilizes the system. Huang et al. (2007) presented an improved IB method. In their study two boundary conditions at the fixed end, i.e., simply supported or clamped, were considered, and it was found that the amplitude of the oscillating filament with the clamped boundary condition is smaller than that with the simply supported boundary condition. Connell and Yue (2007) proposed a fluid–structure direct simulation (FSDS) method to study the flag flapping problem. They identified three distinct states within a range of mass ratio: the static stable state, the period-one limit-cycle flapping state, and the chaotic flapping state. At large Reynolds numbers both the IB method and FSDS method become prohibitively expensive because of the need of very fine grids in the flow field. For the case of an infinite Reynolds number, the panel method has been used to simulate the inviscid and incompressible fluid (Tang et al., 2009a, 2009b). By using the panel method to solve the hydrodynamic pressure and the Galerkin method to solve the dynamics of the flag, Tang and Païdoussis (2007) investigated the post-critical behavior of the flutter system. They found that the critical flow velocity is sensitive to the length of the flag when the flag is short, while it is almost invariant when the flag is sufficiently long. Several periodic flapping states had been observed with the increase of the mass ratio. Alben and Shelley (2008) studied the nonlinear behavior of a two-dimensional flutter system. With the decrease of the dimensionless rigidity, four states were observed in their study: the static stable state, the lower-frequency periodic state, the higher-frequency periodic state, and the chaotic state. A two-dimensional unsteady vortex model based on the Brown–Michael equation was proposed by Michelin et al. (2008). They also carried out a proper orthogonal decomposition (POD) analysis, and it was found that the deformation wave of the flag travels in the streamwise direction, and the forces in the flag travel in the upstream direction.

The chaotic state has been reported in some numerical studies (Connell and Yue, 2007; Alben and Shelley, 2008; Michelin et al., 2008) and experimental studies (Ait Abderrahmane et al., 2011, 2012). But until now the route to chaos has remained unclear. In the present paper, we develop a two-dimensional model to study the dynamics of a flag immersed in an inviscid flow, and examine the route from a static steady state to a chaotic state with the increase of the dimensionless incoming flow velocity. This paper is organized as follows: a model of the coupled fluid–structure system is proposed in Section 2 and validated in Section 3; the route to a chaotic state with the increase of the dimensionless incoming flow velocity is presented in Section 4; and a summary is given in Section 5.

2. Theoretical model and numerical method

2.1. Problem formulation

A flag with a high height-to-width ratio is often considered to be two-dimensional (Eloy et al., 2007; Doaré et al., 2011). Here we considered a model of a two-dimensional flag immersed in flow. In the model, the centerline of the flag is assumed to be inextensible. As shown in Fig. 1(a), the flag is clamped at the leading edge and free at the trailing edge. The length of the flag is L, the mass per unit length of the flag is ρ_s, the second moment of area is I, and the Young's modulus is E. The mass per unit area of the fluid is ρ_f and the undisturbed incoming flow velocity is U.

As shown in Fig. 1, the governing equations of the flag are derived in a local coordinate system. The local tangential direction is denoted by $\vec{\tau}$ and the local normal direction is denoted by $\vec{n}$. According to Newton's 2nd law, the balance of forces on a segment of the flag (Fig. 1(b)) yields

$$\frac{d\vec{F}^i}{ds}+\vec{f}^e-\frac{d}{dt}(\rho_s\vec{V})=0, \tag{1}$$

where s is the distance along the flag from the fixed end, t is the time, $\vec{F}^i$ is the internal force of the flag, $\vec{f}^e$ is the ambient fluid stress acting on the flag, and $\vec{V}$ is the velocity of the flag. The internal force $\vec{F}^i$ includes two components: the tension force in the tangential direction T and the shear force in the normal direction Q.

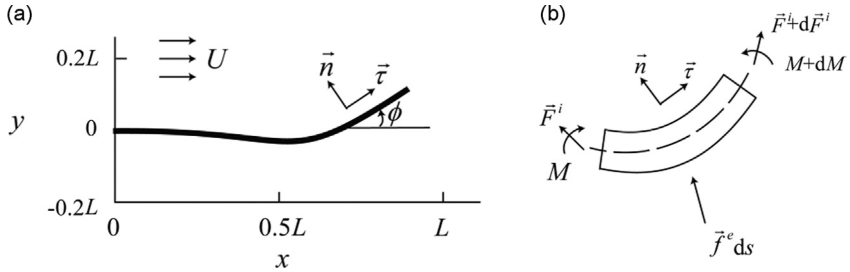

Fig. 1. (a) Description of the problem: a two-dimensional flag immersed in a flow. (b) Forces and moments acting on a segment of the flag.