



Optical solitons to the resonance nonlinear Schrödinger equation by Sine-Gordon equation method

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ABSTRACT

In this paper, we examined the optical solitons to the resonant nonlinear Schrödinger equation (R-NLSE) which describes the propagation of solitons through optical fibers. Three types of nonlinear media fibers are studied. They are; quadratic-cubic law, Kerr law and parabolic law. Dark, bright, dark-bright or combined optical and singular soliton solutions are derived using the sine-Gordon equation method (SGEM). The constraint conditions that naturally fall out of the solution structure which guarantee the existence of these solitons are also reported.

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1. Introduction

Optical solitons have promising potential to become principal information carriers in telecommunication due to their capability of propagating long distance without attenuation and changing their shapes [1]. Nonlinear Schrödinger equations (NLSEs) appear in various areas of engineering sciences, physical and biological sciences. In particular, the NLSEs appears in fluid dynamics, nonlinear optics, plasma and nuclear physics [1,2]. The R-NLSE portrays the dynamics of Madelung fluids in several nonlinear systems [3–9]. We observe many new progresses in the field of nonlinear optics [10–39].

In this study, the R-NLSE is examined. We investigate three types of nonlinear media including quadratic-cubic, Kerr and Parabolic laws in nonlinear fibers. In order to integrate the equation for each type of nonlinearity, the SGEM [10,11] will be employed to achieve this task. This naturally lead to some constraints placed on the soliton parameters and are discussed in their respective sections.

2. Governing equation

The dimensionless form of the R-NLSE that will be studied in this paper is given by Refs. [3–9]:

$$i\psi_t + \alpha\psi_{xx} + \beta F(|\psi|^2)\psi + \gamma \left(\frac{|\psi|_{xx}}{|\psi|} \right) \psi = 0, \quad i = \sqrt{-1}, \quad (1)$$

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where x is the non-dimensional distance along the fiber and t represents the temporal variable. $\psi(t, x)$ is the dependent variable, α is the coefficient of group-velocity dispersion (GVD), β denotes the coefficient of non-Kerr nonlinearity while γ is the coefficient of resonant nonlinearity. The function $F(|\psi|^2)\psi$ is a real-valued algebraic function and is k -times continuously differentiable [12], so that

$$F(|\psi|^2)\psi \in \sum_{m,n=1}^{\infty} C^k((-n, n) \times (-m, m); \mathbb{R}^2). \quad (2)$$

To study Eq. (1), we use the following transformation

$$\psi(t, x) = u(\xi)e^{i\phi(t, x)}, \quad \xi = \lambda(x + vt), \quad (3)$$

where

$$\phi = -kx + \omega t + \theta, \quad (4)$$

where $\phi(t, x)$ represents the phase component, ω is the frequency, k represents the wave number, θ represents the phase constant, v is the velocity and λ is the width of the soliton. Putting Eq. (3) into Eq. (1) and separating the into real and imaginary components, two equations are obtained. The imaginary component gives

$$v = 2k\alpha, \quad (5)$$

and the real part gives

$$(k^2\alpha + \omega)u - \beta F(u^2)u - (\alpha + \gamma)\lambda^2 u'' = 0. \quad (6)$$

3. The sine-Gordon equation method

Consider the following sine-Gordon equation

$$\psi_{xt} = \alpha \sin(\psi), \quad (7)$$

where α is a non-zero constant. Applying the transformation

$$\psi(x, t) = u(\xi), \quad \xi = \eta(x + vt), \quad (8)$$

where v is the traveling wave velocity. Substituting Eq. (8) into Eq. (7)

$$u'' = \frac{\alpha}{v\eta^2} \sin(u(\xi)). \quad (9)$$

Eq. (9) can be simplified to give

$$\left[\left(\frac{u}{2}\right)'\right]^2 = \frac{\alpha}{v\eta^2} \sin^2\left[\frac{u(\xi)}{2}\right] + C, \quad (10)$$

where C is a constant of integration. By letting $C = 0$, $w(\xi) = \frac{u(\xi)}{2}$ and $f^2 = \frac{\alpha}{v\eta^2}$, Eq. (10) reduces to

$$w'(\xi)^2 = f^2 \sin^2(w(\xi)), \quad (11)$$

and in a more simplified form gives

$$w'(\xi) = f \sin(w(\xi)). \quad (12)$$

Setting $f = 1$ in Eq. (12), we get

$$w'(\xi) = \sin(w(\xi)). \quad (13)$$

Eq. (13) has the following solutions

$$\sin(w(\xi)) = \operatorname{sech}(\xi) \quad \text{or} \quad \cos(w(\xi)) = \tanh(\xi), \quad (14)$$

and

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