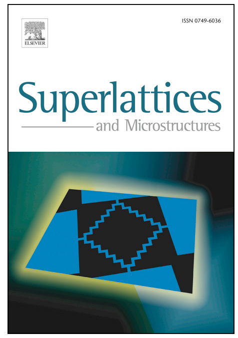

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Dumitru Baleanu, Mustafa Inc, Aliyu Isa Aliyu, Abdullahi Yusuf

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Optical Solitons, Nonlinear Self-Adjointness and Conservation Laws for the Cubic Nonlinear Shrödinger's equation with Repulsive Delta Potential

Dumitru Baleanu[1,2], Mustafa Inc[3], Aliyu Isa Aliyu[3,4], Abdullahi Yusuf[3,4]

[1]Department of Mathematics, Cankaya University, Ankara, Turkey

[2]Institute of Space Sciences, Magurele, Romania

[3]Firat University, Science Faculty, Department of Mathematics, 23119 Elazig/Turkiye

[4]Federal University Dutse, Science Faculty, Department of Mathematics, 7156 Jigawa/Nigeria

E-mails: dumitru@cankaya.edu.tr, minc@firat.edu.tr, aliyu.isa@fud.edu.ng, yusufabdullahi@fud.edu.ng

Abstract

In this paper, the complex envelope function ansatz method is used to acquire the optical solitons to the cubic nonlinear Shr**ö**dinger's equation with repulsive delta potential ($\delta-$NLSE). The method reveals dark and bright optical solitons. The necessary constraint conditions which guarantee the existence of the solitons are also presented. We studied the $\delta-$NLSE by analyzing a system of partial differential equations (PDEs) obtained by decomposing the equation into real and imaginary components. We derive the Lie point symmetry generators of the system and prove that the system is nonlinearly self-adjoint with an explicit form of a differential substitution satisfying the nonlinear self-adjoint condition. Then we use these facts to establish a set of conserved vectors for the system using the general Cls theorem presented by Ibragimov. Some interesting figures for the acquired solutions are also presented.

Keywords: Complex envelope function ansatz, Soliton, Cls, $\delta-$NLSE.

1 Introduction

In the last few decades, studies of soliton propagation in optical fibers have been flourishing because of the rich features of models describing these phenomena. A soliton is a self-localized solution of a nonlinear evolution equation describing the evolution of a nonlinear dynamical system with an infinite number of degrees of freedom [1]. The distinction between solitary wave and soliton solutions is that when any number of solitons interact, they do not change form and the only outcome of the interaction is a phase shift [2]. The existence of solitary wave solutions implies a perfect balance between nonlinearity and dispersion, which usually requires rather specific conditions and can not be established in general [3]. Recently, efficient ansatz have been proposed to construct various types of solutions to NLSEs [4-8]. These solutions include bright, dark and combined solitons with interesting properties. However, several other integration techniques are available to study the dynamics of solitons to nonlinear models [9-45]. The dimensionless form of $\delta-$NLSE that is going to be studied in this paper is given by [9-14]

$$iq_t + \frac{q_{xx}}{2} - \alpha\delta q - \gamma\ |q|^2 q = 0, \quad i = \sqrt{-1}, \tag{1}$$

where $\alpha \neq 0$ and $\gamma \neq 0$ are real constants, δ is the Dirac measure at the origin [9]. The cubic $\delta-$NLSE with localized potential gives a simpler model that describe the resonant nonlinear propagation of light through optical wave [9]. The delta potential is called 'attractive' for $\alpha < 0$, and 'repulsive' for $\alpha > 0$. The stability of the solitons of Eq.(1) is studied by Goodman [9]. Fukuizumi [10] studied Eq.(1) by