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# On a formulation for anisotropic elastoplasticity at finite strains invariant with respect to the intermediate configuration

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## Abstract

The paper is concerned with a formulation of anisotropic finite strain inelasticity based on the multiplicative decomposition of the deformation gradient $\mathbf{F} = \mathbf{F}_e\mathbf{F}_p$. A major feature of the theory is its invariance with respect to rotations superimposed on the inelastic part of the deformation gradient. The paper motivates and shows how such an invariance can be achieved. At the heart of the formulation is the mixed-variant transformation of the structural tensor, defined as the tensor product of the privileged directions of the material as given in a reference configuration, under the action of $\mathbf{F}_p$. Issues related to the plastic material spin are discussed in detail. It is shown that, in contrast to the isotropic case, any flow function formulated purely in terms of stress quantities, necessarily exhibits a non-vanishing plastic material spin. The possible construction of spin-free rates is discussed as well, where it is shown that the flow rule must then depend not only on the stress but on the strain as well.


*Keywords:* Anisotropic plasticity; Orthotropic yield function; Multiplicative inelasticity; Finite strains; Invariance to rotations

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## 1. Introduction

Since its introduction, the multiplicative decomposition of the deformation gradient into an elastic and inelastic part $\mathbf{F} = \mathbf{F}_e\mathbf{F}_p$ (Kröner, 1960; Besseling, 1968; Lee, 1969) became a corner stone in the theory of plasticity. At the crystal level the decomposition describes the slip kinematics and is well defined by the slip directions. At a continuum level the decomposition can be viewed as a non-linear generalisation of the classical additive decomposition of the linear strain tensor into elastic and inelastic parts. Based on it, many continuum formulations have been developed which found also access into finite element-based commercial packages.

At a continuum level, a close inspection, however, reveals that the decomposition is well established only within the realm of isotropic material behaviour. When it comes to anisotropic material behaviour, formulation of inelastic theories based on the mentioned multiplicative decomposition becomes a non-trivial task. The decomposition introduces a so-called intermediate configuration and allows for the distinction of three sets of strain and stress measures: those defined with respect to the actual configuration, those defined with respect to the material configurations, and those defined with respect to the so-called intermediate configuration. Appropriate transformations under the action of $\mathbf{F}$, $\mathbf{F}_p$, or $\mathbf{F}_e$ allow for the translation of one set of stress and strain measures to the other. Now, when utilising the decomposition, the classical approach to formulate an inelastic theory is based on two functions: the free-energy function (or stored energy function) and the flow function. In the isotropic case the two functions can be equivalently defined in terms of any set of stress and strain measures. The flow function, and so the flow rule, can be formulated in terms of the spatial Kirchhoff stress, or, equivalently, in terms of the Mandel stress defined at the intermediate configuration, or, in terms of a purely material Eshelby-like stress. Corresponding inelastic rates exist in all configurations which ensure the physical equivalence whatever the mathematical setting is. Moreover, the isotropy of the functions mean that they are automatically invariant with respect to rotations superimposed on $\mathbf{F}_p$. Under isotropy, the discussion of such an invariance is a non-issue. Also the plastic material spin, within the framework of classical theory, is zero, unless a corresponding adequate evolution equation is defined.

When it comes to anisotropy, the mentioned equivalence can be destroyed and invariance to superimposed rotations is not achieved automatically. It must be imposed, if deemed relevant. Moreover, in such a case the flow function cannot be only a function of the stress alone but must depend on $\mathbf{F}_p$ as well. Depending on the underlying assumptions it can also depend on the strain. All these aspects are completely absent in the isotropic case.

Anisotropic plasticity has been considered in Papadopoulos and Lu (2001) and Schroeder et al. (2002) within the framework of additive decompositions of strain measures, that is without any reference to the multiplicative decomposition. Recently some multiplicative formulations of finite strain plasticity, not invariant in the above sense, have been developed in Eidel and Gruttmann (2003), Itskov and Aksel (2004), and Sansour et al. (2006). Specifically, in the last contribution it was shown that any anisotropic flow function formulated solely in terms of stress quantities will exhibit a non-vanishing plastic material spin.

The idea of invariance as such is motivated by the invariance of the decomposition $\mathbf{F} = \mathbf{F}_e\mathbf{Q}\mathbf{Q}^T\mathbf{F}_p$ for any $\mathbf{Q}$ being an orthogonal tensor. While isotropic formulations inherit