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Optical and other solitons for the fourth-order dispersive nonlinear Schrödinger equation with dual-power law nonlinearity

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ABSTRACT

In this work, the soliton solutions of the fourth-order nonlinear Schrödinger equation (NLSE) with dual-power law nonlinearity is analyzed using Ricatti-Bernoulli (RB) sub-ODE and modified Tanh-Coth methods. We obtain new solutions that are not in existence in previous time. The constraint conditions between the soliton parameters are determined. The solutions we obtained may be used to explain and understand the physical nature of the wave spreads in the most dispersive optical medium.

1. Introduction

The analysis of the NLSE has gotten some remarkable performance for decades, due to its wide spectrum of application. Different types of NLSE are used to explain real phenomena in different domains such as nonlinear optics [1–3], Bose-Einstein deliquescence [4,5], and fluid dynamics [6] and many others. The distinguishing factor between the types of NLSE found is some kind of nonlinearity [7,8] or the terms combined to the equation to generate more light on the perturbation existing when the electromagnetic pulses spread in the optical extreme. Recently, a lot of researches have been committed to the discovery of the traveling wave solutions of many types of NLSE [9–14]. For NLSE, some authors concentrated on the Kerr law nonlinearity media whereby the refractive index is proportional to the light strength [15,16], the fourth-order NLSE and obtained solitary wave solutions for the situation of Kerr law and power law nonlinearity is investigated in Ref. [17]. 1-soliton solution for this situation by soliton ansatz method is presented in Ref. [18], bright and dark soliton solutions for the identical situation using the complex envelope ansatz method and F-expansion method are found in Ref. [19], optical solitons in medium with parabolic law nonlinearity and higher order dispersion [20], 1-soliton in a nonlinear medium with higher-order dispersion and nonlinearities [21] and bright, dark and singular solitons in magneto electro-elastic circular rod [22]. However,

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non-Kerr nonlinearity is regarded at the time the optical materials shows the physical effects which includes maximum absorption. In this work, the non-Kerr law we use is the dual-power law nonlinearity, and it is the general one. Dual-power law is applied to characterize the maximum absorption of the nonlinear refractive index and occurs in the analysis of spatial solitons in photovoltaic-photorefractive materials like $LiNO_3$. Optical nonlinearities in different organic and polymer materials can be modeled by applying this type of nonlinearity [23]. Moreover, discovery of optical and other soliton solutions for NLSE have been presented in the literature using different analytical methods [24–49], these optical solitons are localized electromagnetic waves that spread in nonlinear dispersive media and leave the intensity constant due to the stability between dispersion and nonlinearity effects. These types of solitary waves are vital due to their flexibility in the long-distance optical communication [50–58]. This work focus on the use of the Ricatti-Bernoulli sub-ODE [59,60] and the Modified Tanh-Coth methods [61] to obtain the optical and other soliton solutions of the fourth-order dispersive NLSE with dual-power law nonlinearity and it is given by

$$iu_t + au_{xx} - bu_{xxxx} + c\left(\left|u\right|^{2p} + \varepsilon\left|u\right|^{4p}\right)u = 0. \tag{1}$$

In Eq. (1), $u(x,t)$ is the complex envelope of electromagnetic field, x is the distance along the direction of dissemination, t is the retarded time, a and b are real valued constants and stand on behalf of the second and fourth-order dispersion coefficient, respectively [62], while c is the nonlinearity coefficient. The quantity ε is a constant standing on behalf of the saturation of the nonlinear refractive index, and p is the power law parameter. The fourth-order dispersive NLSE with dual-power law nonlinearity. The NLSE has not recieved more attention as far as we know. Therefore, looking for more soliton solutions of Eq. (1) will be a remarkable contribution.

The rest of the work is organized in the following direction: In Section two, we give the description of the two methods, in section three, we present the application of the two methods and the obtained solutions, in Section four, we give the physical and graphical presentation of the obtained results. Finally, we give a concluding remarks in section five.

2. Description of the methods

2.1. RB sub-ODE method

Consider a nonlinear PDE as follows,

$$P(q, q_t, q_x, q_{xx}, q_{tx}, \ldots) = 0, \tag{2}$$

where P is in general a polynomial function of its arguments, the subscripts denote the partial derivatives. The RB sub-ODE method consists of three steps.

- Step 1. Convert x and t to one variable as follows

$$q(t,x) = q(\xi), \tag{3}$$

and

$$\xi = k(x + Vt), \tag{4}$$

where the localized wave solution $q(\xi)$ travels with speed V, by using Eqs. (3) and (4), one can transform Eq. (2) to an ODE

$$P\left(q, q', q'', q''', \ldots\right) = 0, \tag{5}$$

- Step 2. Assume that Eq. (5) is the solution of the RB equation

$$q' = aq^{2-m} + bq + cq^m, \tag{6}$$

In Eq. (6), a, b, c, and m are constants and will be found later. Taking the second and third derivatives of Eq. (6) yield

$$q'' = ab(3-m)q^{2-m} + a^2(2-m)q^{3-2m} + mc^2q^{2m-1} + bc(m+1)q^m + \left(2ac + b^2\right)q, \tag{7}$$

$$q''' = ab(2-m)(3-m)q^{1-m} + a^2(2-m)(3-2m)q^{2-2m} + m(2m-1)c^2q^{2m-2} + bcm(m+1)q^{m-1} + \left(2ac + b^2\right)q', \tag{8}$$