Journal of Fluids and Structures 24 (2008) 720–731

JOURNAL OF FLUIDS AND STRUCTURES

www.elsevier.com/locate/jfs

Spectrographic analysis for the modal testing of nonlinear aeroelastic systems☆

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Received 2 October 2006; accepted 6 November 2007
Available online 2 January 2008

Abstract

The spectrograph is a signal-processing tool often used for the frequency domain analysis of time-varying signals. When the signal to be analyzed is a function of time, the spectrograph represents the frequency content of the signal as a sequence of power spectra that change with time. In this paper, the usefulness of the technique is demonstrated in its application to the analysis of the time history response of a nonlinear aeroelastic system. The aeroelastic system is modelled analytically as a two-dimensional, rigid airfoil section free to move in both the bending and pitching directions and possessing a rigid flap. The airfoil is mounted by torsional and translational springs attached at the elastic axis, and the flap is used to provide the forcing input to the system. The nonlinear system is obtained by introducing a freeplay type of nonlinearity in the pitch degree-of-freedom restoring moment. The airfoil is immersed in an aerodynamic flow environment, modelled using incompressible thin airfoil theory for unsteady oscillatory motion. The equations of motion are solved using a fourth-order Runge–Kutta numerical integration technique to provide time-history solutions of the response of the airfoil in the pitch and plunge directions. Time-histories are obtained for the nonlinear responses of the linear and nonlinear aeroelastic systems to a sine-sweep input. The time-histories are analyzed using the spectrographic technique, and the frequency content of the response is plotted directly as a function of the input frequency. Results show that the combination of the sine-sweep input with the spectrographic analysis permits a unique insight into the behavior of the nonlinear system with a minimum of testing. It is shown that the frequency of the nonlinear system response is a function of the input frequency and one other characteristic frequency that can be associated with the limit cycle oscillations of the same nonlinear system subject to a transient input.

Keywords: Aeroelasticity; Spectrographic analysis; Nonlinear dynamics; Signal processing; Frequency domain analysis

1. Introduction

Modal testing is often employed in the determination of natural frequencies and damping levels in aircraft structures. In the forced response to a sine-sweep, a time-varying sinusoidal exciting force is applied over a range of frequencies

☆An earlier version of this paper was presented in the 7th FSI, AE & FIV + N Symposium, held within the 2006 PV & P Conference in Vancouver, BC, Canada.

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0889-9746/$ - see front matter
doi:10.1016/j.jfluidstructs.2007.11.003

Nomenclature

a_h	nondimensional distance measured from the airfoil mid-chord to the elastic axis
b	airfoil semi-chord
c_h	translational viscous damping coefficient in plunge
c_α	torsional viscous damping coefficient in pitch
c_β	nondimensional distance of the flap hinge from the airfoil mid-chord
h	plunge displacement of the airfoil
I_α	moment of inertia of the airfoil/flap about the elastic axis
K_h	linear structural stiffness in plunge
K_α	linear structural stiffness in pitch
m	the combined aileron/flap mass per unit span
m_β	the flap mass per unit span
m_0	restoring moment preload for bilinear spring
r_α	nondimensional radius of gyration of the airfoil/flap about the elastic axis
U	nondimensional free stream velocity, $V/b\omega_\alpha$
U^*	nondimensional linear flutter velocity
V	free stream velocity
x_α	nondimensional distance from the airfoil center of mass to the elastic axis
x_β	nondimensional distance from the flap center of mass to the flap hinge
α	pitch rotation of the airfoil, measured about the elastic axis
α_f	α at the start of the freeplay region
β	angular rotation of flap about flap hinge
δ	length of the bilinear stiffness freeplay region
ζ_a	viscous damping ratio in pitch, $c_\alpha/2I_\alpha\omega_\alpha$
ζ_ξ	viscous damping ratio in plunge, $c_h/2m\omega_h$
μ	airfoil–air mass ratio, $m/\pi b^2$
ξ	nondimensional plunge displacement, h/b
τ	nondimensional time, tV/b
ω_ξ	uncoupled frequency ratio, ω_h/ω_α

and the response of the system is measured. Signal processing tools are used to obtain the frequency response function, which may be analyzed to find the natural frequencies, mode shapes and damping values for the aeroelastic system. These parameters can be used to predict the responses to various excitations and to improve the dynamic behavior of the system through design modification. Commonly used signal processing tools typically assume that the system is linear and the parameters time-invariant. The frequency response function obtained using these tools describes how the system responds to sinusoids at different frequencies, and is used to calculate modal damping values. The calculation of the frequency response function is based on the fact that a sinusoidal input to a linear system gives rise to a sinusoidal output at the same frequency as the input, although not necessarily at the same amplitude or in phase with it.

When these modal analysis techniques are applied to nonlinear dynamical systems, the results can be unexpected and accurate results for system frequency and damping values can be difficult to obtain. Nonlinearities in aeroelastic systems can arise from both structural and aerodynamic sources and may initiate aeroelastic instabilities well below the flutter speed predicted by linear theory. Typical nonlinear responses include limit cycle oscillations (LCOs) or, in some cases, chaotic response. One characteristic feature of nonlinear systems is that a single frequency excitation, such as a sinusoid, produces a multi-frequency response. In the case of a mechanical system, a sinusoidal input will generate a response (generally small) at multiples, or superharmonics, of the excitation frequency, even in systems that contain only slight nonlinearities. In the case of an aeroelastic system, it has been shown (Marsden and Price, 2001) that the introduction of a limited nonlinearity in one degree-of-freedom of a two degree-of-freedom aeroelastic system can cause distortion of the frequency response to a sine-sweep input. It has also been shown that this distortion can contribute to errors in the values of modal damping and frequency obtained using the frequency response function. These results provide the motivation for research towards a better understanding and predictive ability of the nonlinear content of aeroelastic response signals.

The spectrographic analysis technique is a signal-processing tool used in applications where the signal is not time-invariant, and where the frequency content as a function of time is important. Time–frequency spectrographic analysis is used in speech recognition and rotating machinery condition-monitoring applications (Cohen, 1989; Rohrbaugh, 1993). The technique has been used (Trickey et al., 2002) to study the frequency of LCOs as a function of airspeed. In the present study, the time-varying parameter is the frequency of the input force, allowing the input frequency/output frequency relationship over a range of frequencies to be studied using a single test at each airspeed.