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Application of the fully automatic 3D *hp* adaptive code to orthotropic heat transfer in structurally graded material properties

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Abstract

The prime objective of the paper is the application of the fully automatic *hp* adaptive Finite Element Method (FEM) code to solve heat transfer problems over the domain with various materials. Large changes in material data generate singularities: non-continuities or large gradients of the solution to the heat equation. To minimize the error of FEM solution the mesh should be refined close to the singularities by sub-dividing elements and increasing the polynomial order of approximation. It is very difficult to design an optimal *hp* mesh by hand. This paper presents a typical application of the fully automatic 3D *hp* adaptive FE code to the problem of modelling of the resistance heating of an Al–Si billet in a steel die for thixoforming process. The code automatically produces an FE mesh with minimal number of degrees of freedom, resolving all singularities within a prescribed error tolerance.

Keywords: Finite Element Method; *hp* adaptivity; resistance heating; thixoforming

1. Introduction

In the modelling of the heat transfer problem, the object under consideration may consist of various materials characterised by various properties. The singularities of the solution occur at interfaces between different materials. Large error of the numerical solution is observed in those areas. For elliptic problems, the error propagates into the entire domain. To minimize the error of the solution, the finite element (FE) mesh must be refined. There are five main mesh adaptive strategies [1]:

- Uniform h refinement, where all finite elements are broken into 4 sons in 2D and 8 sons in 3D.
- Uniform p refinement, where the polynomial order of approximation is uniformly raised over the entire mesh.
- h adaptivity, where only some finite elements are broken, into 2 or 4 sons in 2D, or 2, 4 or 8 sons in 3D, over the mesh areas with large error estimations.
- p adaptivity, where the polynomial order of approximation is increased only on some edges, faces and interiors in 3D, over the mesh areas with large error estimations.
- *hp* adaptivity, where only some finite elements are broken and the polynomial order of approximation p is increased only in areas with large error estimations. This is p adaptivity with h adaptivity added.

The *hp* adaptive strategy is the fastest mesh refinements strategy delivering exponential convergence of the numerical error with respect to the number of degrees of freedom used [1]. General environments to support local mesh refinements have been developed [2–4]. The objective of this paper is presentation of capabilities of the fully automatic *hp* adaptive code described in Refs. [5–8] as far as improvement of the accuracy and decrease of the computational costs are considered. Problem of modelling of the resistance heating of Al–Si billet in steel die (Fig. 1), which is used for thixoforming processes, was selected. The particular goal of the paper is to show how the application of the *hp* adaptivity can reduce size of the FE mesh required to solve the problem with prescribed accuracy.

2. Problem formulation

Model used in this paper is based on the resistance heating of Al–Si billet in steel die for thixoforming process, presented in Ref. [9]. The orthotropic heat equation

$$-\nabla^T \mathbf{k} \nabla T + Q = 0 \tag{1}$$

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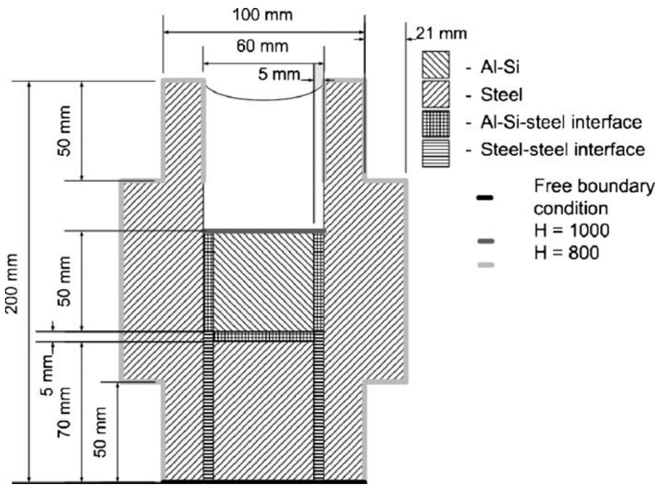

Fig. 1. Geometry and boundary conditions.

Table 1
Material properties

Material	(a) Heat generation Q (W/m^3)	(b) Thermal conductivity k (W/mK)	(c) Boundary convection H (W/m^{2}K)
Al–Si	2000	160	1000
Steel	100000	45	800
Al–Si interface	2000	8	–
Steel–steel interface	100000	5	–

is solved over the domain presented in Fig. 1. Here **k** denotes the matrix of thermal conductivity coefficients, T is the temperature and Q is the heat generated in a volume as a result of electrical current. The generated heat is balanced with heat convection on model boundaries. This is the simplified approach assuming that the heat is constant over each part of the domain, see Table 1 column (a). There are three parts of the assembly: Al–Si billet, steel die and steel stamp. Interfaces between these parts are introduced as "artificial materials". The thermal conductivity k coefficients are presented in Table 1 column (b). The Fourier boundary condition of the third type

$$\frac{\partial T}{\partial n} = H(T_{\text{env}} - T) \tag{2}$$

is defined on the domain boundary, except the bottom of the domain, where the free boundary condition is assumed. Here H is a boundary convection coefficient presented in Fig. 1 and in Table 1 column (c), and T_{env} is the ambient temperature.

3. Description of the fully automatic *hp* adaptive algorithm

The automatic *hp* adaptive code starts from the initial mesh presented in Fig. 2, called the coarse mesh. Various degrees of grey denote various orders of polynomial approximation. The code performs global *hp* refinement of the coarse mesh, and the fine mesh, presented in Fig. 2, is obtained by breaking each finite element from the coarse mesh into 8 sons and increasing the polynomial order of approximation by one. The code solves the problem twice, on the coarse mesh and on the fine mesh. The relative error estimation for the coarse mesh solution is calculated:

$$e_{\text{rel}} = \frac{||u_{h,p} - u_{h/2,p+1}||_1}{||u_{h/2,p+1}||_1} \tag{3}$$

where $u_{h,p}$ is the FE solution on the coarse mesh, $u_{h/2,p+1}$ the FE solution on the fine mesh and $|| \, ||_1$ is the norm in the H^1 Sobolev space (the energy norm). Then, for each finite element from the coarse mesh the code considers all possible refinement strategies. For each possible refinement of each coarse mesh element, the corresponding local solution is obtained by computing projection from the fine mesh solution [5]. Then, the relative error estimation over the coarse mesh element is calculated for each refinement possibility, and the code selects the optimal refinement, giving maximum rate of the error decrease, defined as the ratio of the relative error estimation to the number of the added degrees of freedom. The optimal mesh after the first iteration, presented in Fig. 2, becomes the coarse mesh for the second iteration, and the entire process is repeated, as it is presented in Fig. 3, as long as the relative error estimation over the coarse mesh (3) is greater than the prescribed error tolerance.

In order to be able to mix elements with various size and polynomials with various orders of approximation over element edges, faces and interiors, the following mesh regularity rules, described in detail in Refs. [5–8], are introduced:

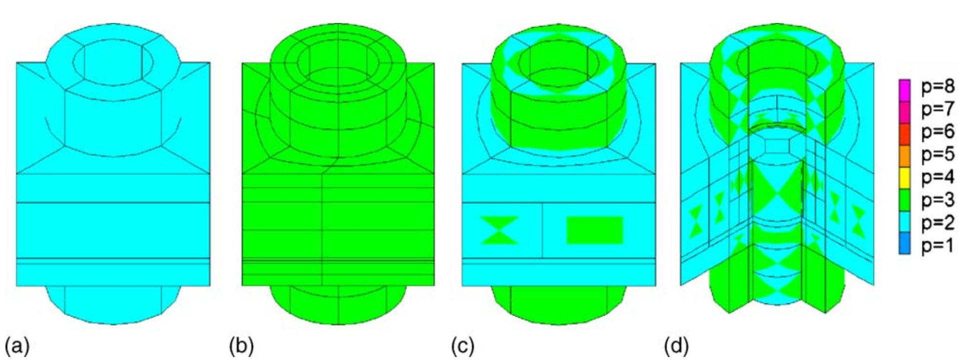

Fig. 2. (a) Initial coarse mesh. (b) Fine mesh. (c) Optimal mesh after the first iteration. (d) Interior of the optimal mesh. Various degree of grey denotes various polynomial orders of approximation p over elements edges and faces.