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Development of a new quadratic shell element considering the normal stress in the thickness direction for simulating sheet metal forming

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Abstract

The existing degenerated shell element based on Mindlin–Reissner's theory of plates assumes that the thickness of the element remains constant and that the normal stress in the thickness direction is zero. These assumptions lead to problems in simulating sheet metal forming, especially in predicting the amount of springback. In this paper, we present a new non-linear shell element by considering the normal stress in the thickness direction for plain strain analysis, which allows for large deformation. Two pseudo nodes are introduced on the top surface and bottom surface, respectively. As a result, the incremental normal strain in the thickness direction can be calculated directly and the incremental normal stress in the thickness direction can be calculated using the constitutive relation directly. Thus, it is possible to deal with the problem that pressure acts on the top surface and the bottom surface is simultaneously constrained. Numerical examples show that the stress prediction is improved. The normal stress in the thickness direction exhibits a correspondence with boundary loads and the bending moment decreases as the normal stress in the thickness direction increases. This indicates the element can simulate the effect of bottoming, i.e. sheet metal is stamped further with a considerable punch force after the punch reaches the dead point in the process of V-type bending for the purpose of reducing springback.

Keywords: Sheet metal forming; Non-linear finite element analysis; Shell element; Thickness change

1. Introduction

Springback is a type of shape distortion in sheet metal, which occurs when the sheet metal is removed from the die after forming because of elastic deformation. It is a major forming defect. In order to stamp sheet metal into the designed shape, it is necessary to predict springback accurately. Because there are problems in the calculation model and the material model, etc., it is very difficult to predict springback accurately. Since springback angle is determined by the internal residual bending stress after forming and the residual bending stress depends on the deformation history, accurate calculation of stress distribution in the forming stage is critical to the accuracy of springback simulation. Due to faster calculation speed, the degenerated shell element is usually used to simulate the sheet metal forming process. The conventional degenerated shell element is based on Mindlin–Reissner's theory of plates with transverse shear deformations included [1,2], which uses the following assumptions [3,4]:

- The line that is normal to the mid-surface in the initial configuration remains straight.
- Normal stress in the thickness direction is neglected.
- The thickness of the element remains constant.

The degenerated shell element has performed well in structural analysis. However, problems arise when it is used to simulate sheet metal forming, especially in predicting

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springback. The second approximation leads to the following two problems. First, the bending stress is overestimated and springback is greater than that measured by experiment since bending stress decreases as normal stress in the thickness direction increases when the equivalent stress reaches the yield point. Second, the 3d constitutive relation cannot be used directly. The third approximation is not true when high pressure acts on both surfaces.

There are several promising approaches that can be used to overcome the problems of the degenerated shell element. The first group (see [5,6]) is based on a solid element and makes modifications that permits stretching through out the thickness. The second group (see [7,8]) is based on Mindlin–Reissner's shell model (with three displacement degrees of freedom at the mid-surface and two rotational degrees of freedom of the fiber). This second approach adds a number of parameters to account for stretching throughout the thickness. The third group (see reference [9]) uses a similar approach but circumvents the use of rotation parameters.

In this research, we took another approach. We introduced two pseudo nodes on the top surface and bottom surface, respectively. This makes it possible to calculate the normal strain in the thickness direction and thus allow for the direct use of 3d constitutive relations. It is possible to treat the problem that the top surface of the sheet is stamped by the punch while the bottom surface of the sheet is simultaneously constrained by the die. In order to satisfy the pure bending conditions, the selective reduced integration (SRI) scheme is adopted with reduced integration for normal strain in the thickness direction and shear strain. The SRI actually falls within the concept of mixed finite element methods as shown by Malkus and Huges [10].

The FEM code for the new-proposed non-linear shell element is newly developed and implemented in the static explicit program ITAS2D, which is FEM software for sheet metal forming developed by the Institute of Physical and Chemical Research. Updated Lagrangian formulation is employed. The code is tested and evaluated in several very important sheet metal forming examples. The simulation results demonstrate that sheet normal stress is compatible with both surface loads. The bending moment decreases as pressure increases when pressure acts on the top surface and the bottom surface is constrained. This suggests that the new-proposed shell element can simulate the effect of bottoming in V-type bending.

2. New shell formulation

2.1. Coordinate systems

The new-proposed quadratic shell element is shown in Fig. 1.

The following coordinate systems are introduced.

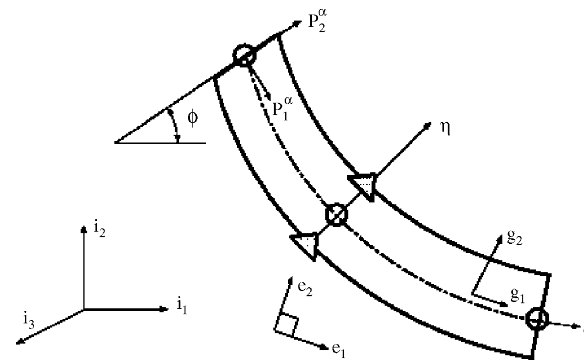

Fig. 1. New-proposed quadratic shell element. Triangle symbol: introduced pseudo node; Circle symbol: displacement node which is same as that of existing degenerated shell element.

1. "shell frame" (or local coordinate system x', y', z), which is on the midsurface.

$$e_1 = \frac{g_1}{|g_1|}, \quad e_3 = (0 \quad 0 \quad 1), \quad e_2 = e_3 \times e_1$$

where

$$g_i = \frac{\partial x}{\partial \xi_i}, \quad i = 1, 2$$

2. P_1^α, P_2^α, P_3^α, which are orthogonal coordinate system at the nodal point.

$$P_2^\alpha = \frac{g_2}{|g_2|}, \quad P_3^\alpha = (0 \quad 0 \quad 1), \quad P_1^\alpha = P_2^\alpha \times P_3^\alpha$$

2.2. Interpolation functions

The coordinates of a point in the shell element are:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \sum_{\alpha=1}^{3} N^\alpha(\xi) \left\{ \begin{pmatrix} x^\alpha \\ y^\alpha \end{pmatrix} + \frac{1}{2}\eta t^\alpha \begin{pmatrix} P_{21}^\alpha \\ P_{22}^\alpha \end{pmatrix} \right\} \tag{1}$$

where α denotes the node number, $N^\alpha(\xi)$ the shape function of node α in the midsurface, x^α and y^α the coordinates of node α, t^α the length of the fiber of node α, $P_{21}^\alpha = \cos\phi^\alpha$; $P_{21}^\alpha = \sin\phi^\alpha$ and ϕ^α is the angle between the fiber of the node and the x axis at node α.

The velocity of a point v_i can be expressed by [11]

$$v_1 = \sum_{\alpha=1}^{3} N^\alpha(\xi) \left\{ v_1^\alpha - \frac{1}{2}\eta t^\alpha P_{11}^\alpha \dot{\theta}_3^\alpha \right\} \tag{2}$$

$$v_2 = \sum_{\alpha=1}^{3} N^\alpha(\xi) \left\{ (1-\eta^2) v_2^\alpha - \frac{1}{2}\eta t^\alpha P_{12}^\alpha \dot{\theta}_3^\alpha \right\} + \sum_{\alpha=4}^{5} N^\alpha(\eta) v_2^\alpha \tag{3}$$

where

$$N^1 = \frac{1}{2}\xi(\xi-1), \quad N^2 = \frac{1}{2}\xi(\xi+1), \quad N^3 = 1-\xi^2,$$

$$N^4 = \frac{1}{2}\eta(\eta-1), \quad N^5 = \frac{1}{2}\eta(\eta+1)$$