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Mathematical model of the formation of a gas hydrate on the injection of gas into a stratum partially saturated with ice☆

I.K. Gimaltdinov, M.K. Khasanov

Ufa State Petroleum Technological University, Ufa, The Sterlitamak Branch of Bashkir State University, Sterlitamak, Russia

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ABSTRACT

The injection of a cold gas into a porous medium saturated with gas and ice that is accompanied by the formation of a gas hydrate is studied theoretically. Self-similar solutions describing the distribution of the basic parameters in the stratum are constructed for an axisymmetric problem with an extended region of phase transitions. The possible existence of four different types of solutions is indicated according to which, when cold gas is injected, it is possible that either a hydrate is formed from gas and ice on the front surface or there is hydrate formation from gas and ice in one extended region or hydrate formation both from gas and ice and gas and water in two extended regions or ice melts on the front surface and a hydrate is subsequently formed from gas and water on another front surface. Critical diagrams are constructed that distinguish the regions of existence of the different types of solutions.

The considerable interest in the investigation of flows in porous media accompanied by phase transitions is due to the need for a theoretical study of the large number of issues arising, for example, in solving problems concerning the extraction of hydrocarbon resources. In particular, many processes in the oil and gas industry are accompanied by the formation of gas hydrates and are usually of a negative character causing emergency shut-downs of the plant on account of the deposition of hydrates both in the stratum as well as in the underground and above-ground equipment systems of oil and gas deposits. However, hydrate formation can also play a positive role and, for example, it can be used for the conservation of gas in the gas hydrate state that allows the capacity of underground gas storage reservoirs to be increased[1] since, under identical conditions, considerably more gas is contained per unit volume of a gas hydrate than it is in the free state.[2]

Several principles of the application of the methods of the mechanics of multiphase media for the mathematical modelling of seepage processes accompanied by phase transitions have been formulated in Refs 3 and 4. Mathematical models of the decomposition of gas hydrates in a porous medium based on the equations of the mechanics of multiphase media have been presented[5–8] and solutions have been obtained for the case when phase transitions occur on the front surface which only provide an adequate mathematical description for a limited range of values of the parameters characterizing the system state and the intensity of the action in the stratum.

Mathematical models of the injection of a cold gas into a porous stratum partially saturated with water, which is accompanied by gas hydrate formation, have been constructed.[9–12]

However, in choosing suitable geological objects for the conservation of gas in the form of a gas hydrate, a porous medium partially saturated with ice is more promising. The important advantage of this system lies in the fact that the intensity of the hydrate formation process in highly permeable porous media is mainly limited by the rate of heat removal and the specific heat of formation of a gas hydrate from ice is three times lower than when it is formed from water.[1]

It has been established [13] as a result of an experimental investigation of gas hydrate formation in a porous medium from ice and water that the accumulation of a gas hydrate in a porous space actively occurs not only in wet porous media but also in rocks partially saturated with ice. Here, the rate of hydrate formation falls off more rapidly in the first case than in the second case.

Unlike in the papers in Refs 5 – 12 where the model of the seepage of a gas accompanied by the formation of a hydrate from gas and water and the decomposition of a hydrate into gas and water is proposed in a linear-parallel approximation, an analytical solution of the

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E-mail address: hasanovmk@mail.ru (I.K. Gimaltdinov).

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axisymmetric problem of the injection of gas into a porous medium partially saturated with ice and for the case when phase transitions occur in an extended region is constructed below.

1. Statement of the problem and basic equations

We assume that a horizontal porous stratum of constant thickness and unbounded extent is saturated with gas and ice at the initial instant. The roof and bottom of the stratum are impermeable and thermally insulated. A gas of the same kind as the initial gas is injected into a hole the opens up onto the whole thickness of the stratum. As a result, the process of the conversion of the gas and ice into a gas hydrate begins.

We make the following assumptions in the mathematical modelling of heat and mass transfer processes in a porous medium accompanied by hydrate formation. The temperatures of the porous medium and the saturating substance (gas, hydrate or ice) are identical. A hydrate is a two-component system with a mass concentration of gas G. The skeleton of the porous medium, the gas hydrate and the ice are incompressible and fixed, the porosity is constant and the gas is calorifically ideal:

$$\rho_{\text{sk}}, \rho_h, \rho_i, m = \text{const}, \quad p = \rho_g R_g T$$

Here, $\rho_j (j = \text{sk}, h, i, g)$ are the true densities of the gas, m is the porosity, p is the pressure, T is the temperature, R_g is the gas constant and the subscripts sk, h, i and g refer to the parameters of the skeleton of the porous medium, the hydrate, the ice and the gas respectively.

Taking account of the assumptions made, we write the equations for the conservation of the masses of the gas and ice in the axisymmetric approximation[4]

$$\frac{\partial}{\partial t}\left(mS_g\rho_g\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(rS_g m\upsilon_g\rho_g\right) = -mG\rho_h\frac{\partial S_h}{\partial t} \tag{1.1}$$

$$\frac{\partial}{\partial t}\left(mS_i\rho_i\right) = -m(1-G)\rho_h\frac{\partial S_h}{\partial t} \tag{1.2}$$

$$S_g + S_i + S_h = 1 \tag{1.3}$$

where $S_j (j = g, i, h)$ is the saturation of the pores of the j-th phase and υ_g is the gas phase velocity.

The seepage of the gas obeys Darcy's law

$$mS_g\upsilon_g = -\frac{k_g}{\mu_g}\frac{\partial p}{\partial r} \tag{1.4}$$

where k_g and μ_g are the permeability and the dynamic viscosity of the gas phase.

The relation between the gas permeability coefficient k_g and the gas saturation can be given on the basis of Kozeny's formula[14]

$$k_g = k_* \frac{(mS_g)^3}{(1-mS_g)^2} \approx k_0 S_g^3 \qquad (k_0 = k_* m^3)$$

The heat flux equation without taking the barothermal effect into account is written in the form

$$\rho c\frac{\partial T}{\partial t} + \rho_g c_g m S_g \upsilon_g \frac{\partial T}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda\frac{\partial T}{\partial x}\right) + m\rho_h L_h\frac{\partial S_h}{\partial t}$$

$$\rho c = (1-m)\rho_{sk}c_{sk} + m\sum_{j=g,l,h} S_j\rho_j c_j, \quad \lambda = (1-m)\lambda_{sk} + m\sum_{j=g,l,h} S_j\lambda_j \tag{1.5}$$

Here, L_h is the specific heat of hydrate formation, ρc and λ are the specific volumetric heat capacity and the thermal conductivity coefficient of the system, c_j and λ_j are the specific heat capacity and the thermal conductivity coefficient of the phases. Since the skeletal parameters of the porous medium make the greatest contribution to the ρc and λ values, we will assume that they are constant in the whole of the stratum.

The temperature and pressure values in the hydrate formation region are related by the phase equilibrium condition[2]

$$T = T_0 + \theta\ln\left(p/p_{s0}\right) \tag{1.6}$$

where T_0 is the initial temperature of the system, p_{s0} is the equilibrium pressure corresponding to the initial temperature and θ is an empirical parameter that depends on the type of gas hydrate.

We assume that three characteristic zones arise in a stratum when the cold gas is injected into the porous medium saturated with the gas and ice: a near zone (the first zone) where the pores are filled with gas and hydrate, an intermediate zone (the second zone) in which gas, ice and hydrate are simultaneously present, and a distant zone (the third zone) that is filled with gas and ice. A hydrate is formed in the intermediate zone. Correspondingly, two surfaces arise: the interface between the distant and intermediate zones where the hydrate