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On the forms of loss of stability of a cylindrical shell under an external side pressure☆

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ABSTRACT

Two statements of the problem of the static loss of stability (through flexural-shearing and non-classical beam flexural modes) of a cylindrical shell made out of an isotropic material under the action of an external pressure on its side surface are considered. The first statement corresponds to the introduction of an assumption concerning the invariability of the direction of the action of the load when the shell changes from the unperturbed equilibrium state to a perturbed equilibrium state and the second statement corresponds to the action of a "tracking" hydrostatic pressure on the shell. The linearized equations of the theory of stability obtained earlier are used with a refinement of the terms in them associated with the action of the "tracking" external pressure and which are written in the linear approximation in the two versions. Exact analytical solutions are obtained for all the formulated problems. The solutions from them describing the beam flexural buckling modes (BMs) correspond to a hinged mounting of the ends of the shell, and they are obtained both on the basis of simplified equations, formulated in the momentless approximation, as well as the unsimplified (moment) equations. It is shown that their realization is only possible in the case of long shells under the action of a "non-tracking" side pressure with a fully explicable loss of stability mechanism. Under the action of a "tracking" pressure, the solutions for beam flexural BMs apparently only exists on account of the asymmetry with respect to the circumferential coordinate of the "loading" terms that occur in the neutral equilibrium equations used when the exact expression for the vector of the unit normal to the deformed middle surface of the shell is replaced by approximate linearized expressions for the vector of the normal that does not have a unit length. The critical values of the pressure corresponding to such beam flexural BMs are considerably greater than the critical pressure corresponding to the "shell" flexural BMs of the shell, and they are therefore of no practical interest whatsoever and only of purely theoretical interest.

The results of investigations of the stability of cylindrical shells made out of isotropic constructional materials that have been carried out by numerous authors over several decades are contained, in particular, in a monograph,[1] and the results for shells made out of composite materials are contained in the monographs in Refs 2 and 3 etc. and are mainly concerned with classical flexural BMs.[1] Together with them, it has been established[4–6] that a number of other (non-classical) BMs exist in the case of cylindrical shells which can be realized earlier than the classical flexural BMs under some or other forms of loading and certain combinations of the determining physico-mechanical and geometric parameters of the shell. One of these non-classical BMs is beam bending that can be realized in long shells under the action of an external pressure and is characterized by the retention, on loss of stability, of the circular shape of the cross section. In a physical sense and according to the geometric pattern of the realization, it is similar to the flexural BM of a straight rod under conditions of compression in a transverse direction that was recognized and investigated in detail in Ref. 7. In the case of an elongated cylindrical shell, the solution of the linearized problem for this BM has been found[8] starting out from the maximally simplified equations, obtained by reducing the two-dimensional linearized equations of the momentless theory[4] to the one-dimensional equations of the theory of rods on the basis of the well-known Timoshenko shear model, that are formulated under the assumption that the initial (subcritical) direction of the action of the external pressure is retained when it passes into a perturbed state. In this paper, the same problem is considered on the basis of both the

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linearized moment equations and the linearized momentless equations which have a greater degree of accuracy and richness of content compared with those used previously. It is formulated in two statements, the first of which, as previously,[7,8] is based on the introduction of the above mentioned assumption concerning the retention of the direction of the action of the load and the second statement corresponds to the action of a "tracking"[9] hydrostatic pressure on the shell.

1. The two versions of the statement of the problem

We will consider a thin cylindrical shell with a thickness t, radius of the middle surface R and length L that is under the action of an external pressure p. It is assumed that the middle surface σ of the shell is related to the axial coordinate x $(0 \le x \le L)$ and the circumferential coordinate θ $(0 \le \theta \le 2\pi)$ and we denote the unit vectors directed along the tangents to the coordinate lines, which, together with the vector of the unit normal $\mathbf{m}$ to the surface σ constitute a right-handed trihedron, by $\mathbf{e}_1$ and $\mathbf{e}_2$. If the elongation parameter of the shell $\lambda = R/L$ satisfies the conditions $\lambda \ll 1$, then the circumferential force T_{22}^0 formed in it can be determined to a high degree of accuracy using the formula

$$T_{22}^0 = -pR \tag{1.1}$$

Together with the initial stress state characterized by the equality (1.1), we will consider an adjacent equilibrium state, and the change to this state at a certain value p_* called the critical value, occurs due to an increase in the vector of the displacements of points of the middle surface σ by an amount

$$\mathbf{u} = u\mathbf{e}_1 + \upsilon\mathbf{e}_2 + w\mathbf{m}$$

For arbitrary displacements and strains, the vector of the unit normal $\mathbf{m}_*$ to the deformed middle surface σ_* is determined by the expression[10,11] (ε_{ij} are the components of the shear strain tensor of the surface σ)

$$\begin{gathered}\mathbf{m}_* = I^{-1/2}\left(E_1\mathbf{e}_1 + E_2\mathbf{e}_2 + E_3\mathbf{m}\right)\\ E_1 = e_{12}\omega_2 - (1+e_{22})\omega_1, \quad E_2 = e_{21}\omega_1 - (1+e_{11})\omega_2, \quad E_3 = (1+e_{11})(1+e_{22}) - e_{12}e_{21}\\ I = 1 + 2I_1 + 4I_2, \quad I_1 = \varepsilon_{11} + \varepsilon_{22}, \quad I_2 = \varepsilon_{11}\varepsilon_{22} - \varepsilon_{12}^2\end{gathered} \tag{1.2}$$

Here (a derivative with respect to a coordinate x or θ is denoted by a subscript after a comma)

$$\omega_1 = w_{,x}, \quad \omega_2 = \frac{w_{,\theta} - \upsilon}{R}, \quad e_{11} = u_{,x}, \quad e_{22} = \frac{\upsilon_{,\theta} + w}{R}, \quad e_{12} = \upsilon_{,x}, \quad e_{21} = \frac{u_{,\theta}}{R} \tag{1.3}$$

In setting up the neutral equilibrium equations intended for studying the BMs of a shell, by virtue of the smallness of the strain and displacement increments, expression (1.2) is taken in the linearized approximation. Following the known results,[9] we write it in the form (the underlined terms will be discussed below)

$$\mathbf{m}_* = -\omega_1\mathbf{e}_1 - \omega_2\mathbf{e}_2 + \left(1 + \underline{e_{11} + e_{22}}\right)\mathbf{m} \tag{1.4}$$

corresponding to the introduction of the assumption that $I \approx 1$ which is valid for small strains when the estimates $\omega_i \sim \sqrt{\varepsilon}$, $e_{ij} \sim \sqrt{\varepsilon}$, where ε is a certain quantity that is small compared with unity, are satisfied. We note that the length of the vector (1.4), unlike the vector (1.2), is not equal to unity.

By definition,[9] the external pressure p is not a tracking pressure if the surface force vector $\mathbf{X}$, given in the initial state by the equality

$$\mathbf{X} = -p\mathbf{m} \tag{1.5}$$

preserves its direction of action on changing from the initial state to the adjacent perturbed state and is called a "tracking" pressure if the equality

$$\mathbf{X} = -p\mathbf{m}_* = p[\omega_1\mathbf{e}_1 + \omega_2\mathbf{e}_2 - (1 + \underline{e_{11} + e_{22}})\mathbf{m}] \tag{1.6}$$

holds in the perturbed state.

In the case of intermediate bending of the shell when the estimates $\omega_i \sim \sqrt{\varepsilon}$, $e_{ij} \sim \varepsilon$, hold, instead of (1.4) the simpler expression[10] $\mathbf{m}_* = -\omega_1\mathbf{e}_1 - \omega_2\mathbf{e}_2 + \mathbf{m}$ is used which enables us in the case of a tracking pressure to take the representation

$$\mathbf{X} = p\left(\omega_1\mathbf{e}_1 + \omega_2\mathbf{e}_2 - \mathbf{m}\right) \tag{1.7}$$

instead of (1.5).

Relations based on the so-called improved Timoshenko shear model (Refs 2,3,10–12 etc.) are one of the most common versions of the relations of the shells theory at the present time and, according to this model, the approximation

$$\mathbf{U} = \mathbf{u} + z\gamma = (u + z\psi)\mathbf{e}_1 + (v + z\chi)\mathbf{e}_2 + (w + z\gamma)\mathbf{m}, \quad -t/2 \le z \le t/2 \tag{1.8}$$

where ψ and χ are function of the rotations of cross sections of the shell x = const and θ = const and γ is a function of the transverse compression, is taken for the displacement vector in the direction of the normal $\mathbf{m}$.

A consistent version of the geometrically non-linear equations of the shell theory of general form, that is based on the model (1.8) and does not allow appearances of "false" bifurcation solutions of problems on shell stability, has been constructed earlier.[6] If the transverse shear strains are assumed to be constant throughout the shell thickness,[10,11] then, in approximation (1.1) in the system of linearized neutral