Journal of Applied Mathematics and Mechanics 80 (2016) 73–83

Contents lists available at ScienceDirect

Journal of Applied Mathematics and Mechanics

journal homepage: www.elsevier.com/locate/jappmathmech

Contact problems of the sliding of a punch with a periodic relief on a viscoelastic half-plane☆

I.G. Goryacheva, A.P. Goryachev

A. Yu. Ishlinskii Institute for Problems in Mechanics of the Russian Academy of Sciences National Research Nuclear University, Moscow, Russia

ARTICLE INFO

Article history:
Received 14 July 2015
Available online 16 June 2016

ABSTRACT

A method is proposed for the analytical solution of problems of the sliding of a punch with a periodic relief on a viscoelastic half-plane under conditions when they are in complete contact as well as problems on the loading of a viscoelastic half-plane with a piecewise-constant pressure that moves at a constant velocity on its surface. The problems are considered in a quasistatic formulation. The method is based on the expansion of the sought functions in trigonometric Fourier series and finding the relation between the displacements of the boundary and the contact pressures. Examples of different shapes of punches are considered and the dependencies of the contact characteristics and the mechaical component of the friction force on the sliding velocity, the viscosity parameters of the base and the geometric characteristics of the surface are studied.

© 2016 Elsevier Ltd. All rights reserved.

When there is friction in elastomers which include resins, rubbers, viscoelastic polymers and composite materials based on them, one of the main sources of energy dissipation is elastic hysteresis that occurs during the cyclic deformation of the surface layers of the materials. Models of viscoelastic bodies[1–4] are used to describe this process. Another factor affecting the friction force to a considerable degree is the microgeometry of the contacting surfaces. In fact, it determines the frequency of the interaction of the material and the time during which it comes into contact with the counter body. The study of the combined action of these factors is important in developing techniques for controlling the friction forces in the contact of bodies possessing imperfect elasticity of the surface layers.

A number of aspects of this problem have been studied in a quasistatic formulation.[5–14] Contact problems in a plane formulation concerning the motion of a periodic system of punches on a thin viscoelastic layer bonded to an elastic half-plane have been considered.[5–8] The deformations of the layer were described by one-dimensional Maxwell [6,7] and Kelvin[8] models. The three-dimensional contact problem of the sliding of a system of spherical irregularities on a viscoelastic layer described by a one-dimensional Kelvin model has been investigated.[9] Contact problems of the sliding of a doubly periodic wavy surface on a viscoelastic layer described by a one-dimensional Kelvin model and characterized by a relaxation time spectrum[10,13,14] have been investigated with the aim of studying the effect on the contact characteristics and friction force not only of the shapes of the vertices of the irregularities but also of the indentations in the rough surface. The case of complete[10] and discrete[11] contact have been considered. A sliding contact of a doubly periodic wavy surface on a viscoelastic layer when there is an incompressible fluid in the gap has been investigated[12] and the effect of adhesion in the gap between a wavy surface and a viscoelastic layer described by a one-dimensional Kelvin model in a plane [11] and spatial [14] formulation has been studied.

1. Constitutive relations for a viscoelastic half-plane

We take the relations between the strain components ε_{x0}, ε_{y0}, γ_{x0y0} and the stress components σ_{x0}, σ_{y0}, τ_{x0y0} in anisotropic viscoelastic medium in the following form [15] (the plane deformation case):

☆ *Prikl. Mat. Mekh.*, Vol. 80, No. 1, pp. 103–116, 2016.
E-mail address: goryache@ipmnet.ru (A.P. Goryachev).

http://dx.doi.org/10.1016/j.jappmathmech.2016.05.011
0021-8928/© 2016 Elsevier Ltd. All rights reserved.

$$\varepsilon_{x^0} + T_\varepsilon \frac{\partial \varepsilon_{x^0}}{\partial t} = \frac{1-\nu^2}{E}\left(\sigma_{x^0} + T_\sigma \frac{\partial \sigma_{x^0}}{\partial t}\right) - \frac{\nu(1+\nu)}{E}\left(\sigma_{y^0} + T_\sigma \frac{\partial \sigma_{y^0}}{\partial t}\right)$$
$$\varepsilon_{y^0} + T_\varepsilon \frac{\partial \varepsilon_{y^0}}{\partial t} = \frac{1-\nu^2}{E}\left(\sigma_{y^0} + T_\sigma \frac{\partial \sigma_{y^0}}{\partial t}\right) - \frac{\nu(1+\nu)}{E}\left(\sigma_{x^0} + T_\sigma \frac{\partial \sigma_{x^0}}{\partial t}\right)$$
$$\gamma_{x^0 y^0} + T_\varepsilon \frac{\partial \gamma_{x^0 y^0}}{\partial t} = \frac{1+\nu}{E}\left(\tau_{x^0 y^0} + T_\sigma \frac{\partial \tau_{x^0 y^0}}{\partial t}\right) \tag{1.1}$$

where T_ε and T_σ characterize the viscous properties of the medium ($T_\varepsilon > T_\sigma$), E and ν are the constant Young's modulus and Poisson's ratio and $H = T_\varepsilon E / T_\sigma$ is the instantaneous modulus. Equations (1.1) are a two-dimensional analogue of the one-dimensional Kelvin model. In the case of a uniform motion of the punch with a velocity V on the boundary of the half-plane, the stress and strain distribution in it can be assumed to be steady with respect to the system of coordinates associated with the punch: $x = x^0 - Vt$, $y = y^0$. In this system of coordinates, the displacements and stresses are explicitly independent of the time and are only functions of the coordinates (x,y). We introduce fictitious stresses and strains using the formulae (see Ref. 15)

$$\varepsilon_{ij}^0 + T_\varepsilon \frac{\partial \varepsilon_{ij}^0}{\partial t} = \varepsilon_{ij} - T_\varepsilon V \frac{\partial \varepsilon_{ij}}{\partial x} = \varepsilon_{ij}^*, \quad \sigma_{ij}^0 + T_\sigma \frac{\partial \sigma_{ij}^0}{\partial t} = \sigma_{ij} - T_\sigma V \frac{\partial \sigma_{ij}}{\partial x} = \sigma_{ij}^* \tag{1.2}$$

The introduced functions satisfy equations that are equivalent to equilibrium equations, the equations of the compatibility of the strains and to Hooke's law for an isotropic elastic body.

It follows from formulae (1.2) that the fictitious displacement $u^*(x)$ and the fictitious pressure $p^*(x)$ are related to the true displacement $u(x)$ of the deformed boundary of the viscoelastic body in the direction of the Oy axis perpendicular to the boundary of the half-plane and the pressure $p(x)$ acting on it by the relations

$$u^*(x) = u(x) - T_\varepsilon V u'(x), \quad p^*(x) = p(x) - T_\sigma V p'(x) \tag{1.3}$$

respectively (a prime denotes differentiation with respect to x).

2. Statement and method of solving the periodic problem for an elastic half-plane

We consider, in a plane quasistatic formulation, the problem of a punch with a periodic contour $f(x) = f(x + l)$ that slides at a constant velocity V on the boundary of a viscoelastic half-plane in the direction of the Ox axis. The following conditions are satisfied on the boundary of the half-plane: In the contact regions $u'(x) = f'(x)$ and, outside the contact regions, $p(x) = 0$. There are no shear stresses in the boundary of the half-plane.

When the punch slides on the viscoelastic base, a linear (acting per unit length of the indentor along the Oy axis) friction force T acts along the direction of the Ox axis due to the asymmetry in the contact pressure, and this force can be calculated using the formula[8]

$$T = \int_0^l p(x) f'(x) dx = \int_0^l p(x) u'(x) dx$$

In deriving this relation, account has been taken of the fact that the amplitude h of the periodic relief is much smaller than its period l ($h \ll l$).

For a uniform motion of a punch on a viscoelastic base, a shear force that compensates for the friction force has to be applied to it. Moreover, the punch is pressed to the base by a force P per unit length that acts for one period and compensates for the contact pressure $p(x)$:

$$P = \int_0^l p(x) dx.$$

We solve the problem of determining the relation between the displacement $u(x)$ of the deformed boundary of the viscoelastic half-plane and the pressure $p(x)$ acting on it as well as the problem of determining the friction force T in a dimensionless form for which we introduce the following dimensionless quantities and parameters:

$$\bar{x} = \frac{x}{l}, \quad \bar{t} = \frac{t}{l}, \quad \bar{p}(\bar{x}) = \frac{2(1-\nu^2)}{\pi E} p(\bar{x}l), \quad \bar{p}^*(\bar{x}) = \frac{2(1-\nu^2)}{\pi E} p^*(\bar{x}l)$$
$$\bar{u}(\bar{x}) = \frac{u(\bar{x}l)}{l}, \quad \bar{u}^*(\bar{x}) = \frac{u^*(\bar{x}l)}{l}, \quad \bar{P} = \frac{2(1-\nu^2)}{\pi E l} P = \int_0^1 \bar{p}(\bar{x}) d\bar{x}$$
$$\alpha = \frac{T_\varepsilon}{T_\sigma}, \quad \beta = \frac{h}{l}, \quad \zeta = \frac{l}{2VT_\varepsilon}, \quad \bar{T} = \frac{2(1-\nu^2)}{\pi E l} T = \int_0^1 \bar{p}(\bar{x}) \bar{u}'_{\bar{x}}(\bar{x}) d\bar{x} \tag{2.1}$$

Download English Version:

https://daneshyari.com/en/article/794827

Download Persian Version:

https://daneshyari.com/article/794827

Daneshyari.com