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On the homogenization of periodic beam-like structures

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Abstract

A homogenization method for periodic beam-like structures that is based on the unit cell force transmission modes is presented. Its main advantage is that to identify the principal vectors of the state transfer matrix corresponding to the transmission modes it operates directly on the sub-partitions of the unit cell stiffness matrix and allows to overcome the problems due to ill-conditioning of the transfer matrix. As case study, the Pratt girder is considered. Closed form solutions for the transmission modes of this girder are achieved and used into homogenization. Since the pure bending mode shows that the Pratt unit cell transmits two kinds of bending moments, one given by the axial forces and the other originated by nodal moments, the Timoshenko couple-stress beam is employed as substitute continuum. Finally, a validation of the proposed procedure is carried out comparing the predictions of the homogenized models with the results of a series of girder f.e. analyses.

Keywords: Pratt and Vierendeel girders; Beam like lattice; Timoshenko couple-stress beam; Homogenization; Force transmission modes; Transfer state matrix eigen-analysis

1. Introduction

Periodic beam-like structures offer the optimal trade-off between strength and stiffness, joined with lightness, economy and manufacturing times. For this reason, they are receiving growing interest from researchers and technicians of several engineering areas and find frequent applications in civil and industrial buildings, naval, aerospace, railways and bridge constructions, material design and bio-mechanics (Salmon et al. (2008); Cao et al (2007); Salehian et al (2006); Cheng et al (2013); Tej and Tejová (2014); Fillep et al (2014); Zhang et al (2016); El Khoury et al (2011); Syerko et al (2013); Ju et al (2008); Kerr (1980); Pucillo (2016); De Iorio et al (2014a - c); De Iorio et al (2017)).

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Modelling these structures with a 1-D homogenized continuum model is of great utility in the real problems. While several micropolar models have been reported for the analysis of planar lattices and periodic micro-structures (Noor (1988); Bazant and Christensen (1972); Kumar and McDowell (2004); Ostoja-Starzewski et al (1999); Segerstad et al (2009); Donescu et al (2009); Warren and Byskov (2002); Onck (2002); Liu and Su (2009); Dos Reis and Ganghoffer (2012); Hasanyan and Waas (2016), to cite a few), the studies on the micro-polar models for beam-like lattices have not yet achieved the same advances. As far as the authors are aware, only few papers have specifically addressed this topic (Noor and Nemeth (1980); Salehian and Inman (2010); Romanoff and Reddy (2014); Gesualdo et al (2017)).

In this work, a method for the homogenization of periodic beam-like structures is reported. It is based on the unit cell state transfer matrix eigen-analysis. This technique so far has been applied mostly for the dynamic analysis of repetitive or periodic structures (Mead (1970); Meirowitz and Engels (1977); Zhong and Williams (1995); Langley (1996)). Only recently, it has also been used for the elasto-static analysis of prismatic beam-like lattices with pin-jointed bars (Stephen and Wang (1996); Stephen and Wang (2000); Stephen and Ghosh (2005)). Its practical implementation is problematic since the state transfer matrix $\mathbf{G}$ is defective and ill-conditioned. To overcome ill-conditioning, in Stephen and Wang (2000) two approaches, the force and displacement transfer methods, are presented. By them, a better conditioning is achieved analysing the behaviour of a lattice of n identical cells.

The method we propose instead operates directly on the sub-partitions of the unit cell stiffness matrix for searching the unit principal vectors of $\mathbf{G}$ and consequently avoids all the numerical drawbacks of the transfer methods till now proposed. For the simple case of the Pratt girder, closed form solutions for the unit cell force transmission modes are obtained and used to evaluate the stiffnesses of the equivalent Timoshenko micropolar beam. The accuracy of the homogenised medium in reproducing the behaviour of real discrete beam-like structures is finally assessed with a sensitivity analysis carried out by finite element models.

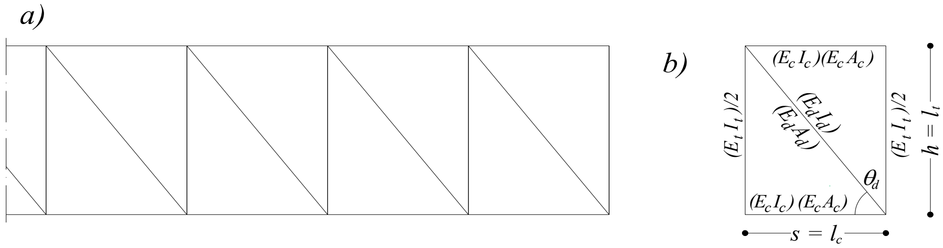

Fig. 1 - Pratt girder (a) and the corresponding unit cell (b)

2. Pratt girder transmission modes

The unit cell of the analysed girder is made up of two straight parallel chords rigidly connected to the webs (see Fig. 1). All the cell members are Bernoulli-Euler beams. The top and bottom chords have the same section whose area and second order central moment are denoted A_c and I_c. To simplify the analysis, the girder transverse webs are assumed axially inextensible. This is equivalent to neglect the transverse elongation among the chords during girder deformation. The cross-sectional area and the second order moment of the diagonal members are A_d and I_d, while I_t denotes the second order moment of the transverse webs. To account the girder periodicity, the two vertical beams of the unit cell will have second order moment equal to the half part of I_t.

To identify any quantity related to the girder *i-th* nodal section, the sub-script *i* will be adopted, see Fig. 2. To distinguish between the joints or nodes of the same section, the superscripts *t* or *p* are used, depending on whether the top or bottom chord is involved. Finally, in a coherent manner, top and bottom nodes of the section *i* are labelled i_t or i_b.

The static and kinematical quantities of the *i-th* cell are schematically shown in Fig. 2. However, for our purposes, it is more convenient to adopt static and kinematic quantities alternative to the standard ones of Fig. 2. More precisely, the deformed shape of the cell will be defined in terms of the mean axial displacement $\hat{u}_j = 1/2\left(u_j^t + u_j^b\right)$, the section rotation $\psi_j = \left(u_j^b - u_j^t\right)/l_t$, the transverse displacement v_j and, finally, the symmetric and anti-symmetric parts of the