

XXVII International Conference “Mathematical and Computer Simulations in Mechanics of Solids and Structures”. Fundamentals of Static and Dynamic Fracture (MCM 2017)

Influence of boundary conditions on stiffness properties of a rectangular nanoplate

Anatolii Bochkarev^a

^a*Saint Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg, 199804 Russia*

Abstract

This paper presents the state equations of the large deflection of a plate in the framework of the classical nonlinear von Kármán theory with surface stresses taken into account. Unlike the original Gurtin–Murdoch model, the present model does not include any non-strain terms in the surface stress–strain relation. It is shown that the classical structure of the potential energy of a deformed nanoplate is preserved with the redefined elastic moduli, which contain the characteristics of a body and a surface, are introduced. This allows applying to nanoplates the known solutions and methods for macroplates. So the difference between the compressive buckling of a macroplate and of a nanoplate is put in evidence, as well as which factors influence this.

Copyright © 2017 The Authors. Published by Elsevier B.V.
Peer-review under responsibility of the MCM 2017 organizers.

Keywords: surface stresses, nanoplate, effective properties, von Kármán

1. Von Kármán plate model with surface stresses

At the beginning of the last century, Gibbs showed that surface tension is present at the boundary between dissimilar materials. A general model of surface stresses as well as surface elasticity has been developed by Gurtin and Murdoch in Gurtin and Murdoch (1975, 1978) and can be reduced to the case of a negligibly thin layer adhering to the underlying material without slipping, and the elastic moduli for both are different. Somewhat later, an atomistic model in Miller and Shenoy (2000), Shenoy (2005) has supported the idea that this continuum surface model (the GM model) can predict fundamentally the same elastic response.

Recently, in problems of nanomechanics, the two-dimensional theories of plates and shells with surface stresses taken into account have been widely used. The surface stresses with the complete GM equation for large deflections of nanoplates have been investigated by the von Kármán plate theory in Lim and He (2004)

* Corresponding author. Tel.: +7 812 428 7179.
E-mail address: a.bochkarev@spbu.ru

and then have been made more precise in Huang (2008), with the effect of non-zero normal stress taken into account in Lu, et al. (2006). However, the original GM surface stress–strain relation includes non-strain terms (the displacement gradient), and therefore averaging the bulk phase and surface stresses over the thickness leads to non-symmetric membrane forces. Considering that this theory itself is of an evaluative nature, a departure from the classical structure of the equations of the plane elastic problem imposes severe restrictions on their use in studying nonlinear effects, even with the homogeneous static boundary conditions. So, the authors of Lim and He (2004); Lu, et al. (2006); Huang (2008) have confined themselves to the study of the one-dimensional case for modeling the bending, buckling, and vibration of an infinite strip. Thereafter, many other authors have been looking for ways to bring the structure of the resolving equations with surface stresses taken into account to the classical one.

In two-dimensional models, the widely used approach is the transition to the simplified GM equation (without the non-strain terms). A discussion of the use of the complete or simplified GM equation was raised in Mogilevskaya, et al. (2008) and continues to this day. This path looks promising because it preserves the classical structure of both the resolving equations and the static boundary conditions.

The GM equation without the non-strain term has allowed the wide use of the effective material properties at the nano- and microscales considering surface effects in the linear theory of plates and shells with transverse shear Altenbach, et al. (2010) and Eremeev (2016). Also the GM equation without the gradient term was used in the description of bending and free vibrations on the basis of the first-order shear deformation theory Ansari, et al. (2014) but without a detailed consideration of the membrane forces. Similarly, by using a strain-consistent model of surface elasticity, the large deflection of nanoplates with induced residual stress has been investigated in Ru (2016), but a continuity in-plane condition has not been considered. A simplified accounting of the surface stresses has been made at the edges of a plate, solving the plane elastic problems in Tian and Rajapakse (2007); Grekov and Yazovskaya (2014), and at the facial surfaces of a plate (through the effective flexural stiffness), solving the buckling problem in Bauer, et al. (2014). But this solution has neglected other effective tangential and flexural properties.

Note the work Ansari and Gholami (2016), in which the authors artificially introduce symmetric membrane forces on the basis of the third-order shear deformation theory of Reddy. With this approach, the resolving equations preserve the classical structure. However, the static boundary conditions that are specified for non-symmetric membrane forces, can not be expressed through symmetric ones. Despite such a serious limitation, this approach allows modeling free and forced oscillations under the homogeneous in-plane kinematic boundary conditions.

The goal of the present paper is to show how, with the help of the introduced effective moduli, the classical form of the state equations and the potential energy of a deformed nanoplate with surface stresses taken into account can be preserved. This opens the possibility of using the well-developed plane theory of elasticity with the effective in-plane properties for the continuum modeling of nanoplates. The results of the application of the constructed theory are shown in the example of the study of the compression buckling of a rectangular nanoplate in comparison with the known classical results for macroplates.

1.1. Equations of state and geometrical relations

Let us consider a homogeneous, linear elastic plate occupying the area $\{(x_1, x_2, z) \in \Omega \times [h/2, +h/2]\}$, $\Omega \subset \mathbf{R}^2$. The Cartesian coordinate system is x_1, x_2 in-plane and z orthogonal to the midplane of the plate ($\mathbf{n}$ is the unit normal). Following Altenbach, et al. (2010), Ru (2016) etc., on the facial surfaces of the plate ($z = \pm h/2$) the linearized constitutive equation Gurtin and Murdoch (1975, 1978) can be expressed as

$$\boldsymbol{\tau}_{\pm} = \tau_0 \mathbf{A} + 2\mu_0^s \boldsymbol{\varepsilon}_{\pm} + \lambda_0^s \mathbf{A} \operatorname{tr} \boldsymbol{\varepsilon}_{\pm}, \quad \mu_0^s = \mu^s - \tau_0, \quad \lambda_0^s = \lambda^s + \tau_0; \quad (1)$$

$$(\tau_{1z}, \tau_{2z})_{\pm} = \tau_0 \nabla w_{\pm}. \quad (2)$$

Download English Version:

<https://daneshyari.com/en/article/7955050>

Download Persian Version:

<https://daneshyari.com/article/7955050>

[Daneshyari.com](https://daneshyari.com)