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On Elementary Theory of Tangent Stresses at Simple Bending of Beams

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Abstract

The article contains three additions to the elementary theory of flexural shear stresses developed by the author, which is a generalization of Zhuravsky's theory. First, we derive a formula for the cross-sectional shape coefficient, which takes into account the deplanation of the cross section in Mor's integrals for energy and displacements. The new form factor, in contrast to the classical one, depends on the Poisson ratio and the ratio of the cross-sectional dimensions. Secondly, a very simple formula is given expressing the potential potential energy of deformation of the rod, connected with the vertical tangential stress. Thirdly, this formula is used for the energy analysis of the author's theory, that establish the new properties of Zhuravsky's. It is stated that, for certain values of the Poisson's ratio (of its own for each type of cross section), Zhuravskii's theory yields exact results that coincide with the results of the theory of elasticity.

Keywords: beam, bending, Zhuravsky's theory, generalization.

1. Introduction

In Kharlab V. D. (2015); Kharlab V. D. (2015), the author proposed *an elementary theory of tangent stresses at simple bending of beams* that generalizes and specifies the theory of Zhuravsky D.I., which is presented in courses on the strength of materials. This article contains three supplements to Kharlab V. D. (2015); Kharlab V. D. (2015): 1) cross-section shape factor; 2) formula of new type for potential energy of deformation; 3) application of the minimum potential energy.

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The Zhuravsky's theory includes an assumption on uniform distribution of the vertical tangent stress τ_{xz} by the cross-section width and the well-known Zhukovsky's formula for this stress. In the middle of the 19th century, the Zhuravsky's theory was an outstanding achievement in the strength of materials, however, now it is in need of flaws correction, which, according to the theory of elasticity, are: wrong above assumption, absence of the formula for the horizontal tangent stress and the fact that significant influence of Poisson's effect remained unaccounted.

Finally, our generalization of the Zhuravsky's theory is as follows:

$$\tau_{xz}(y,z) = \overline{\tau}_{xz}(z)[1-k+12ky^2/b^2(z)], \tag{1}$$

where y, z – main central coordinates of cross-section points; $\overline{\tau}_{xz}(z)$ – stress as per Zhuravsky; $b(z)$ – a cross-section width in the point under consideration; k – constant parameter determined according to the condition of stress entirety using the formula

$$k = \frac{\Phi - (1+\nu)^{-1}}{\Phi + \Psi}, \tag{2}$$

where ν – Poisson's ratio.

$$\Phi = -\langle [\frac{S(z)}{b(z)}]'' \rangle, \;\; \Psi = 24\langle \frac{S(z)}{b^3(z)} \rangle, \tag{3}$$

where $S(z)$ – a static moment of the cut-off cross-section area, and the angle parenthesis mean averaging by points of the cross-section line of symmetry:

$$\langle f(z) \rangle = \int_z f(z)b(z)\,dz / A, \tag{4}$$

A – cross-section area. The Poisson's effect is taken into account through factor k. For example, for a rectangular, circle and triangle, the theory is as follows, respectively:

$$k_п = \frac{\nu}{1+\nu}\frac{1}{1+2(h/b)^2}; k_к = -\frac{1-2\nu}{8(1+\nu)}; k_т = -\frac{1-2\nu}{2(1+\nu)}\frac{1}{1+6(h/b)^2}, \tag{5}$$

where h, b – cross-section height and width.

The formula for the horizontal tangent stress τ_{xy} was derived from the differential equilibrium equation and the result (1) (this formula is not given here).The expression in the square brackets (1) is a correction factor for the Zhuravsky's formula. Kharlab V. D. (2015) contains several methods for determination of factor k*,* the one of them is considered here. The important theory consequence is the formula expressing stresses at points of the cross-section line of symmetry:

$$\tau_{xz}(0,z) = (1-k)\overline{\tau}_{xz}(z). \tag{6}$$

The comparison with the known accurate solutions of problems has shown that the theory under consideration correctly represents the influence of the Poisson's ratio and relation of cross-section dimensions, and in any case it has a reasonably good obtainable accuracy.

Let's start considering the new results.

Cross section shape factor α. As known, it takes into account cross-section distortion in the Mohr integrals for