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Journal of Materials Processing Technology 194 (2007) 7–14

Journal of Materials Processing Technology

www.elsevier.com/locate/jmatprotec

Analytical modeling of wall thinning during corner filling in structural tube hydroforming

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Received 11 December 2006; received in revised form 5 March 2007; accepted 16 March 2007

Abstract

In a tube hydroforming process, an internal pressure is used to expand a circular tube into one with non-circular sections. Friction at the material/die interface plays an important role in determining the distribution of the thickness during deformation. In this paper, an analytical model is developed to determine the variation in the stresses and strains along the tube wall as the internal pressure increases to expand the circular tube into a square cross-section. Based on the analytical model, a parametric study is conducted on the effects of the coefficient of friction and the material properties on the distribution of cross-section thickness in the hydroformed tubes. Such parametric study is useful in establishing guidelines for designing the structural tube hydroforming process. It also helps in the search for limits of the process capabilities. One of the strengths of analytical modeling is that it gives more insights into the mechanism through which friction affects the deformation during tube hydroforming.

Keywords: Tube hydroforming; Corner filling; Thickness variation; Friction coefficient

1. Introduction

Tube hydroforming is a process used to form tubular sections of various geometries from initially circular tubes. The process allows for weight reduction through more efficient section design and improved structural strength and stiffness [1–4]. Tubes with variable cross-sections can be hydroformed in two ways: tube crushing, or closed die tube expansion [5]. In tube crushing, die closure is utilized to assist deformation in corner filling. In closed die expansion, the tube is formed between two die halves through the use of internal fluid pressure. During the deformation, part of the section is in contact with the die. If the material/die interface were frictionless, the strain would be uniformly distributed all over the section and a section with a constant thickness would be formed. Friction at the material/die interface restricts the flow of material in contact with the die and causes non-uniform distribution of strain. It causes a variation in the thickness of the formed tube along the cross-section with the smallest thickness at the corners. Thus the frictional behavior at the material–die interface is crucial in tube hydroforming.

Various studies using analytical models and finite element method have been carried out on tube hydroforming. Xia [6] developed an analytical model to predict bursting and wrinkling failures for the expansion of circular tubes using internal pressure and end feeding. Li et al. [7] categorized hydroforming processes by the stress state and analyzed the effect of stress state on the deformation during tube bulging. This categorization was based on the relation between internal pressure and end feeding during hydroforming. It is helpful in analyzing the tendencies for wrinkling or tearing during forming. Analytical equations were derived to calculate axial feeding forces and the forming pressure in a closed die tube expansion based on force equilibrium [8].

Analytical and experimental studies of the effect of material properties and friction coefficient on bursting during tube bulging were carried out by Carleer et al. [9]. The analytical model predicts that higher *r*-value, higher *n*-value, and lower friction coefficient lead to higher expansion ratios before failure and to more uniform thickness distribution. The effect of the *r*-value is the largest, followed by friction, then the *n*-value.

Chen et al. [10] used finite element simulation to study the effects of internal pressure and coefficient of friction on corner filling in tube hydroforming. Kirdli et al. [11] used finite element simulations to study the effect of material strain hardening, friction coefficient, and die corner radius on the thickness distri-

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0924-0136/$ – see front matter © 2007 Published by Elsevier B.V.
doi:10.1016/j.jmatprotec.2007.03.112

bution. The two-dimensional hydroforming of circular tubes into rectangular sections was studied by Hwang and Altan [12] using FEA. Two processes were compared. In the first a smaller tube was expanded in a closed rectangular die. In the second, a larger tube was crushed into the die before the application of internal pressure. It was found that in the later case a much lower pressure was required to achieve corner filling. The thickness distribution after forming was more uniform for the crushed tube. The same could be concluded for tube hydroforming into triangular section as shown in Hwang and Altan [13]. Experimental measurements of the coefficient of friction during hydroforming were carried out by Vollertsen and Plancak [14]. Their approach depended on correlating thickness variation along the tube axis to friction coefficient when upsetting an internally pressurized tube.

This paper develops an analytical model for the expansion of a circular tube into a square die. Analytical models allow for efficient parametric study to investigate the effect of different process variables on the formed parts. In the second section of this paper, equations that governs the material flow during corner filling are derived from geometrical relations, force equilibrium, and material properties. In the third section, the derived equations are solved for different forming conditions in order to study the effect of friction coefficient and material properties on the quality of the formed tubes.

2. Mathematical model

A quarter model of the process with corner deformation is shown in Fig. 1. In order to built a mathematical model for the deformation during corner forming, a basic understanding of the mechanics of the deformation is necessary. At any given stroke of the deformation the tube section can be divided into two parts, circular corners and straight walls. When a pressure is applied on the tube, lateral forces on the circular corner generate longitudinal forces in the corner. These longitudinal forces in the circular corner are transmitted to the straight part of the tube and stretche the material in that zone. If there were no friction at the tube/die interface, a uniform force will exist along the whole section and uniform thinning will occur along the section. Friction at the tube/die interface dissipates longitudinal force in the straight part causing non-uniform force distribution and thus non-uniform thinning along the section. In order to model the effect of friction, the quarter model of an initially circular tube as shown in Fig. 1 is analyzed as it is subjected to an increasing internal pressure. When internal pressure is applied, the tube deforms such that the corner radius decreases while the contact length between the tube and the rigid die increases. At certain pressure levels a stick zone develops where the material in this zone is not subjected to any further deformation. The size of the stick zone varies with the pressure.

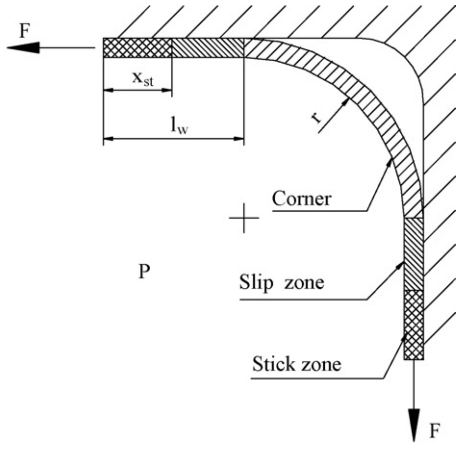

Fig. 1. Model for two-dimensional analysis of tube hydroforming.

In order to simplify the analysis the following assumptions are made:

1. During the deformation, the strain is assumed to be constant across the thickness.
2. Deformation is dominated by stretching. The bending as a result of radius change is neglected.
3. The material is assumed to be rigid-strain hardening with the strain hardening characterized by $\sigma_f = k(\varepsilon_0 + \varepsilon)^n$.
4. The effect of lateral pressure on the flow stress is neglected.
5. A plane strain condition is assumed, with no strain along the length of the tube.
6. The material is assumed to be rate independent rigid-plastic.
7. At any instant, the strain distribution along the corner is uniform, where “corner” refers to the circular part of the tube that is not in contact with the die, see Fig. 1.

These assumptions reduce the problem to a two-dimensional one. No variation in stress or strain is considered along the tube thickness. They impose some limitations on the applicability of the analysis in some cases. In this type of forming processes dominated by stretching, materials with high formability and low friction coefficient are required to achieve reasonable expansion before failure. If not, the size of expanded area before failure is rather small affecting the accuracy of the solution. Variation of the stress across the thickness is not considered in the analysis. The effect of bending at the corner zone is neglected. This reduces the accuracy of the analysis when hydroforming tubular sections of small corner radius to thickness ratios, typically $R/t < 5$. The material model used is based on rate independent plasticity. The model cannot be used to analyze the deformation during the tube hydroforming of materials with high strain rate sensitivity.

As with most metal forming processes the strain state of the material is path dependent. The model has to be solved incrementally. The independent variable in the analysis is the internal pressure. At each increment of the pressure, changes in geometrical and material variables are calculated and used to update their corresponding variables.

This process for developing the analytical model is as follows. The internal pressure needed to initiate yielding in the circular tube is calculated first. The pressure is then increased by dP until the final pressure is reached. As the pressure is increased, the force transmitted from the corner to the wall increases by dF_c. The corner radius changes by dr. The corner strain increases by dε_c. The total stretching in the straight part is given by dl_w. The purpose of the following analysis is to relate these incremen-