Computational Materials Science 110 (2015) 295–301

Contents lists available at ScienceDirect

Computational Materials Science

journal homepage: www.elsevier.com/locate/commatsci

Comprehension of the ferromagnetic hysteresis via an explicit function

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ARTICLE INFO

Article history:
Received 1 December 2014
Received in revised form 13 July 2015
Accepted 10 August 2015
Available online 5 September 2015

Keywords:
Magnetic hysteresis loop
Magnetic induction
Magnetization
Applied magnetic field
Explicit function

ABSTRACT

The paper developed computation method to predict the behavior of hysteresis loops via an explicit function. Based on the Fermi–Dirac statistics and assumption of magnetic domains as fermions, formulations to successfully explain magnetic hysteresis loop are devoted. The capability of the new model to calculate the characteristics of magnetic hysteresis loop was tested by comparing the experimental results of both the variation of magnetic induction with magnetic field intensity and variation of magnetization along magnetic field intensity. The results of the application of the new model confirm the reliability of the procedure. The differences between the calculated and the measured data are always less than the data tolerance usually declared by the experiments. Particular attention to the influences of temperature on the parameters in the explicit function results in determination of magnetization of fixed ferromagnetic material by both magnetic field and temperature. Besides, the paper shows that coercively of ferromagnetic materials can be indicated by temperature. The technique given in this paper is found to be very advantageous for understanding the hysteretic problems.

1. Introduction

To determine the electromagnetic properties of magnetic materials, many measurements of the magnetic hysteresis loop for magnetic materials were performed under dc, or quasi dc, conditions [1–8]. The benefit of magnetic hysteresis loop has covered the areas of (a) solid state physics/condensed matter physics, (b) general physics, (c) materials science and magnetism/electromagnetism, as well as relevant encyclopedias and physics dictionaries. In order to understand how the magnetic properties of ferromagnetic materials are changing under physical and chemical conditions, hysteresis modeling is of high importance. Over the years, several models are introduced-among the more popular ones are the Jiles–Atherton model [9] and Preisach theory [10]. Jiles–Atherton model and many modifications on Jiles–Atherton model [11–13] are differential equations to simulate hysteresis loop of ferromagnetic materials. For solving most important computational problems connected with the Jiles–Atherton model the Gauss–Kronrod quadrature formula, the Runge–Kutta methods and Matlab implementation of DIRECT algorithm have to be used. The Preisach theory and models based on the Preisach theory [14–16] consist of many relay hysterons connected in parallel, given weights, and summed. Therefore, the applications of Jiles–Atherton model, models based on the Jiles–Atherton model, the Preisach theory and modifications on Preisach theory are not easy works. Of course, one method known as a history-differential model [17] is very complex in mathematics. Although Monte Carlo simulation may be used to explain the hysteresis loops of single-domain particles [18], the physical meaning of the parameters obtained via Monte Carlo method are often not clear. It may be stated that many models are available to investigate hysteretic characteristics of ferromagnetic materials but they tend to be complex and difficult to implement.

Explicit function to calculate hysteresis loop of ferromagnetic materials is of continuing interest because the data are pertinent to the theory of ferromagnetic materials and to the understanding of the properties of ferromagnetic materials. The aim of the present paper is to propose an explicit function able to accurately represent the hysteresis loop of ferromagnetic materials on the basis of the experimental results. The model in form of explicit function is derived from Fermi–Dirac distribution and phenomenological method. The accuracy of the explicit function is checked by comparing the results of the explicit function to experimental results of hysteresis loop of three ferromagnetic materials, the experimental data showed very good agreement with the simulation results. Moreover, another novelty of the work is how hysteresis loop of ferromagnetic materials is controlled by temperature is explained via mathematical way. Hence, the model accuracy is good and can be easily adapted to the requirements of the application.

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http://dx.doi.org/10.1016/j.commatsci.2015.08.019
0927-0256/

2. Methodologies

In the view of classical electrodynamics, the energy produced inside the magnetic moment, $\vec{\mu}$, by magnetic induction, $\vec{B}$, is $\vec{\mu}\cdot\vec{B}$, while, in according with the quantum mechanics, the energy created inside the magnetic moment, $\vec{\mu}$, by magnetic induction, $\vec{B}$, become $M_s\mu gB$, where, M_s is magnetic quantum number and g is Lande factor in this expression. The variation of magnetization, M, along magnetic induction, B, is nonlinear due to the characteristic of magnetic domains existing in ferromagnetic materials. In order to explain magnetization via magnetic induction using quantum statistics, the magnetic domains is suggested to be fermions in this paper, therefore, the Fermi–Dirac distribution should apply to magnetic domains. Fermi–Dirac distribution is usually written as:

$$f(E) = \frac{1}{1+\exp((E-E_f)/kT)} \tag{1}$$

where $f(E)$ is the probability that a particle will have energy E, E_f is Fermi energy, T is absolute temperature, k is Boltzmann constant and its value is $k = 1.38 \times 10^{-23}$ in SI unit.

Because the energy of magnetic domains is proportional to magnetic induction, the useful energy produced by magnetic field in average magnetic domains is

$$E = M\bar{\mu}gB \tag{2}$$

or

$$E = \vec{\bar{\mu}}\cdot\vec{B} = |\vec{\bar{\mu}}||\vec{B}|\cos\alpha \tag{3}$$

where $\vec{\bar{\mu}}$ is average magnetic moment of magnetic domain and α is angle between magnetic field and average magnetic moment of magnetic domain. Since the energy of magnetic domains obtained from magnetic field is proportional to magnetic induction in both quantum mechanics and classical electrodynamics, in this paper, the energy created by magnetic induction in magnetic domains is expressed using classical electrodynamics. Hence, the probability that a magnetic domain may be with energy of $E = \bar{\mu}\cos\alpha B$ via magnetic field is

$$f(E) = \frac{1}{1+\exp((E-E_f)/kT)} = \frac{1}{1+\exp((\bar{\mu}\cos\alpha B - E_f)/kT)} \tag{4}$$

A new form of function (4) is

$$\begin{aligned} f(E) &= \frac{1}{1+\exp((\bar{\mu}\cos\alpha B - E_f)/kT)} \\ &= \frac{1}{1+\exp\left(\left(B-\frac{E_f}{\bar{\mu}\cos\alpha}\right)\Big/\frac{kT}{\bar{\mu}\cos\alpha}\right)} \end{aligned} \tag{5}$$

It is easy to rewrite the probability that a magnetic domain can be with energy of $E = \bar{\mu}\cos\alpha B$

$$f(E) = f(B) = \frac{1}{1+\exp((B-B_0)/B_1)} \tag{6}$$

hence, parameters B_0 and B_1 can be defined as below:

$$B_0 = \frac{E_f}{\bar{\mu}\cos\alpha} \tag{7}$$

$$B_1 = \frac{kT}{\bar{\mu}\cos\alpha} \tag{8}$$

Furthermore, one direct conclusion is that magnetization is proportionate to the probability, $f(E)$, in ferromagnetic materials, therefore, magnetization, M, can be write mathematically as:

$$M(B) = \frac{M_2}{1+\exp((B-B_0)/B_1)} \tag{9}$$

where M_2 is proportional constant, it will be determined via both mathematics and boundary conditions in measurements. Hence, as the magnetic induction, B, approaches very high corresponding to magnetic induction, B, to tends to infinity in mathematics, the probability function $f(E)$ asymptotically to zero:

$$f(E)|_{B\to\infty} = f(B)|_{B\to\infty} = \left.\frac{1}{1+\exp((B-B_0)/B_1)}\right|_{B\to\infty} = 0 \tag{10}$$

When the magnetic induction approaches large enough in the forward direction, there is one saturation magnetization in the magnetization vs. magnetic induction curve, and this saturation magnetization can be known as the right top saturation magnetization, M_{rtsat}:

$$M(B)|_{B\to\infty} = M_{\mathrm{rtsat}} \tag{11}$$

Boundary condition (11) predicts that the magnetization and magnetic induction should be related as:

$$M(B) = M_1 + \frac{M_2}{1+\exp\left(\frac{B-B_0}{B_1}\right)} \tag{12}$$

To define the significance of M_1 and M_2 not only depend on mathematical considerations, but also depend on boundary conditions given by experiment. Hence, another mathematical result is given by (13)

$$\left.\exp\left(\frac{B-B_0}{B_1}\right)\right|_{B\to-\infty} = 0 \tag{13}$$

When the reverse magnetic induction approaches low enough, the experimental results indicative that the magnetization, $M(B)$, goes asymptotically to another constant value know as the left bottom saturation magnetization, M_{lbsat}:

$$M(B)|_{B\to-\infty} = M_{\mathrm{lbsat}} \tag{14}$$

Combining Eqs. (12)–(14) yields

$$M_2 = M_{\mathrm{lbsat}} - M_{\mathrm{rtsat}} \tag{15}$$

$$M_1 = M_{\mathrm{rtsat}} \tag{16}$$

Substitution of Eqs. (15 and 16) into (12) leads to following expressions:

$$\begin{aligned} M(B) &= M(B)|_{B\to\infty} + \frac{M(B)|_{B\to-\infty} - M(B)|_{B\to\infty}}{1+\exp\left(\frac{B-B_0}{B_1}\right)} \\ &= M_{\mathrm{rtsat}} + \frac{M_{\mathrm{lbsat}} - M_{\mathrm{rtsat}}}{1+\exp\left(\frac{B-B_0}{B_1}\right)} \end{aligned} \tag{17}$$

here M_{rtsat} is the saturation magnetization at right top predicted via Eq. (17), M_{lbsat} is the saturation magnetization at left bottom expected by Eq. (17). B_0 is the inflection point magnetic induction. Meanwhile, the average value of the magnetization is fixed at B_0. B_1 is constant of magnetic induction. Magnetization can be successful explained by magnetic induction in forward direction via function (17) because it is derived from the quantum statistics and without any approximation.

When the magnetic induction-dependent magnetization in reverse direction is considered, it is not difficult for one to read:

$$M(B) = M'_{\mathrm{rtsat}} + \frac{M'_{\mathrm{lbsat}} - M'_{\mathrm{rtsat}}}{1+\exp((B-B'_0)/B'_1)}\,[M'_{\mathrm{rtsat}} \to M'_{\mathrm{lbsat}}] \tag{18}$$

The variation of magnetic induction, B, with applied field (magnetic field intensity), H, is also hysteresis curve for a ferromagnetic material, attention should be paid to the mathematical relationship between B and H. Since both curves M against B and B along