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Homogenized elastoplastic response of repetitive 2D lattice truss materials

F. Dos Reis [1], J.F. Ganghoffer *

Laboratoire d'Energétique et de Mécanique Théorique et Appliquée, Université de Lorraine 2, Avenue de la Forêt de Haye, BP 160, F-54504 Vandoeuvre-les-Nancy, France

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The asymptotic discrete expansion method is used to construct the initial yield surface of periodic 2D trusses of beams and the evolution of the yield surface with ongoing hardening. It allows simulating the elastoplastic homogenized response of such lattices subjected to multiaxial loadings. The proposed methodology is quite general, as the representative unit cell includes internal nodes and no assumption of uniform deformation is needed. We determined the effective elastoplastic response for the case of stretching dominated lattices without considering bending effects. This methodology has been implemented in algorithmic format in a dedicated code as a user oriented subroutine in finite element calculations, allowing the analysis of a large variety of new 2D lattices. Applications to the conceived Octagon-Mixed, asymmetric and star-square lattices illustrate the powerfulness of the proposed method. The homogenized stress–strain evolutions over loading unloading uniaxial and biaxial cycles are in good agreement with those obtained by finite element simulations performed over complete 2D lattices.

Table of symbols

We use a vector notation for tensors, which are represented by boldface symbols. A superscript ^{trial} means that we consider the trial function and a subscript ^{n} or $^{n+1}$ refers to step n or $n+1$, while a subscript ^{r} indicates the reference to a reduced matrix.

$\dot{\alpha}^b$	flow rate of the beam b
δ^{ib}	translation parameter of the cell
$[d\mathbf{E}]_n$	strain tensor rate at step n, vectorial form
$[d\mathbf{\Sigma}]_n$	stress tensor rate at step n, vectorial form
$[d\sigma]$	vector of the stress rate $d\sigma^b$ of the beam b
$[d\sigma_e]$	vector of the elastic rate $d\sigma_e^b$ of the beam b
$[d\sigma_\gamma]$	vector of the projection of the trial stress on the yield surface
$E(b)$	end node of the beam b
E_s	elastic modulus of the beam material
$[\mathbf{E}]$	homogenized strain tensor, vectorial form
$[e_e]$	strain vector e_e^b of the beam b
$[e_p]$	plastic slip vector e_p^b of the beam b
$\mathbf{e}^b$	director of the beam b
$\mathbf{e}^{b\perp}$	tangent vector of the beam b
λ_i	local cell coordinates
$\mathbf{e}_i^\lambda$	basis of the lattice coordinates
$[F]$	vector of the forces F^b acting on each beam b of the reference cell
$[f]$	vector of yield conditions f^b of the beam b
g	Jacobian of the change of variables function $\mathbf{R}(\lambda_i)$
$[\gamma]$	vector of the plastic flow rates γ^b of the beam b
$[H]$	diagonal matrix of plastic modulus H^b of beam b
k_l	beam stiffness in traction
k_f	beam stiffness in bending
$[K]$	homogenized stiffness matrix
$[K_t]$	tangent matrix
$[K_p]$	plastic matrix stiffness
$[K_e]$	elastic matrix stiffness
$[K_\gamma]_r$	matrix used for the calculation of $[\gamma]_r$
L^b	length of beam b
$\mathbf{M}^i$	moments at beams extremities
n	number of the beams in a cell
$O(b)$	origin node of the beam b
ϕ^i	rotation at node i
$[Q_p]$	matrix relating the homogenized stress $[\mathbf{\Sigma}]$ to the microscopic beam stress $[\sigma]$
$[Q_e]$	matrix relating the forces vector on a cell $[F]$ to the homogenized strain $[\mathbf{E}]$

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* Corresponding author. Tel.: +33 03 83 59 55 52; fax: +33 03 83 59 55 51.
E-mail addresses: francisco.dos-reis@wanadoo.fr (F. Dos Reis), jean-francois.-ganghoffer@univ-lorraine.fr (J.F. Ganghoffer).
[1] Principal corresponding author.

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ρ^*	relative density
$\mathbf{R}(\lambda_i)$	change of variables function from (λ_i) to (x_i)
$[S]$	compliance matrix
$\mathbf{S}^i$	homogenized force vector associated with direction $\mathbf{e}_i^\lambda$
$\mathbf{T}^b$	force vector on beam b
$[\sigma]$	vector of the stress component σ^b of the beam b
$\overline{\overline{\boldsymbol{\Sigma}}}, [\boldsymbol{\Sigma}]$	homogenized stress tensor, respectively in tensorial and vectorial form
$[\boldsymbol{\Sigma}_n]$	normalized stress tensor by s_0, vectorial form
$[s]$	vector of deformation resistance s^b of the beam b
s_0	initial yield limit conditions of the beams
$[\mathrm{sgn}(\sigma)]_{n+1}^{trial}$	diagonal matrix of the sign of the vector $[\sigma]_{n+1}^{trial}$
t	beam's width
$\mathbf{u}^i$	displacement of node i
ν^i	integer associated with real λ^i
$\mathbf{v}^i$	virtual velocity of node i
$\mathbf{w}^i$	virtual rotational velocity of node i
$[\xi]$	diagonal matrix relation between $[d\sigma_\gamma]$ and $[d\sigma_e]$

1. Introduction

The advent of new means of production, such as rapid prototyping, allows to obtain new structured materials having a complex cellular geometry based on repetitive lattices of beams. This has renewed the interest for the study and optimization of lattices of beams and their homogenized mechanical properties, especially the yield resistance and strain hardening behavior. For example, Zhang et al. [25] have designed two new lattices, called the Si-Square and the N-Kagome. For designers wishing to obtain new custom-oriented materials with specific mechanical properties, one must accompany the development of manufacturing methods by efficient calculation methods borrowing from mechanical homogenization, Doyoyo and Hu [11]. There are several specific applications using the domain of plastic deformation of beams lattice. For example, the energy absorption property of metallic foams, usually modeled by a lattice of beams, are used in case of car crash, Banhart [3].

The material strength of architectured materials has been the topic of several studies related to either their elastic strength, or to their non-linear elastoplastic behavior. The choice of method of analysis of such lattices in the plastic range is especially guided by the nature and type of lattice under consideration. Several classification methods exist in the literature: Deshpande proposed to classify lattices in either stretching dominated or bending dominated lattices Deshpande et al. [7], considering that beams are working either in tension–compression, also known as "direct action mechanism", or in bending, Christensen [4], Mohr [18]. The elastic strength of beam lattices has been considered by various authors: Gibson and Ashby [15] analysed foams (bending dominated,) with the relative density as the dominant criterion Demiray et al. [5], Sullivan et al. [21] and Kim and Al-Hassani [17] with a more numerical or analytical approach, Florence and Sab [14] with an energy homogenization method, Deshpande et al. [6] and Wang and McDowell [23] with various stretching dominated lattices, and Doyoyo and Hu [12] studied the octet lattice. The stretching dominated lattices were proved to be much stronger than bending governed lattices in Deshpande et al. [7]; this raises the interest of this material as a substitute for metallic foams in lightweight structures. More recently, the initial yield surface for 2D truss-lattice materials under biaxial loading was investigated by Alkhader and Vural [1], based on FE analyses and analytical techniques relying on an energy criterion for orthotropic materials. The extended finite element method is used in Zhang et al. [24] for the elastoplastic analysis of periodic truss materials in the small strain regime. Multiscale base functions are constructed to capture the small scale heterogeneities of the unit cells; this local information is then brought to the upper macroscopic scale to perform structural calculations. The mechanical properties of micro-lattice structures subjected to normal stresses are evaluated in Ushijima et al. [22], based on an analytical method relying on classical beam theory. The yield surface is determined under an external biaxial loading state.

The investigation of the stress–strain relationships of beam lattices in the plastic range is more involved. The classical criteria of continuum mechanics do indeed not allow to describe the nonlinearities in the plastic range Fan et al. [13]. The effective behavior of three different lattice materials endowed with cubic symmetry has been studied by means of analytical and numerical techniques in Park et al. [20]. A multiscale finite element method was developed by Zhang et al. [24] to analyse the elastoplastic small strain behavior of 2D periodic lattices. A continuum mechanism based multi-surface plasticity model has been introduced by Mohr [18] to simulate the mechanical behavior of 2D or 3D stretching dominated lattices. This method has been extended later by Fan et al. [13]; this model however relies on an underlying hypothesis of uniform deformation of the cells (see Fig. 10). This hypothesis is not necessary true in the case of lattice with internal nodes (in the unit cell), even if the lattice is stretching dominated.

In the present work, we develop the discrete asymptotic homogenization method for the construction of the initial yield resistance domain and the stress–strain relation accounting for ongoing hardening. As one shall see, the main advantage of the proposed method is its ability to handle lattices presenting a non-uniform deformation due to the existence of internal nodes within the reference unit cell.

This contribution is organized as follows: in Section 2, we briefly recall the background behind discrete homogenization, its adaptation in view of the construction of the initial yield domain, and we expose the set of basic equations for the update of the plastic variables in presence of hardening. Applications and examples that illustrate the proposed methodology are given in Section 3, constructing first the initial yield domain for two classical lattices, the triangle and the hexagonal lattice, with bending or extension as a dominant deformation mode respectively. The obtained homogenized results serve the purpose of validating the calculation method of the initial yield domain. The proposed algorithm is next applied to three lattices exhibiting a non-uniform deformation: the octagon-mixed lattice, the asymmetric lattice and the square-star lattice. The obtained homogenized elastoplastic responses are validated by comparison with finite element simulations performed over entire lattices. Finally (Section 4), a summary of the main results and a few perspectives are given.

2. Theory

2.1. Summary of the discrete homogenization theory

2.1.1. Description and parametrization of the lattice

For a repetitive truss-like material, the beams are parameterized as pictured in Fig. 1. One shall notice that the extremity node $E(b)$ is necessarily associated to the node numbered globally with the set of integers $\lambda^i = \left(\nu^1 + \delta^{1b}, \nu^2 + \delta^{2b}\right)$. In most cases, the extremity node belongs to an adjacent cell, which means that the integers δ^{ib} describing the shift of this node to an adjacent cell belong to the set $\{0, 1, -1\}$ as described in Fig. 2. Note that due to the assumed periodicity, the infinite truss is built from the rep-