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A further study on representative models for calculating the residual stress based on the instrumented indentation technique

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In this paper four representative models (Suresh et al.'s model, Lee et al.'s model, Carlsson et al.'s model and Wang et al.'s model) for calculating the residual stress based on the sharp instrumented indentation technique are compared each other. It is found that all the four models can be expressed as the expanded form of Suresh et al.'s model. Numerical simulations are used to investigate the applicability of these four models for materials with different mechanical properties, and the results show that the accuracy of these models is dependent on both strain hardening exponent (n) and yield strain (σ_y/E). It is also indicated that Suresh et al.'s model is more suitable for materials with a low strain hardening exponent and yield strain; Lee et al.'s model seems more appropriate to materials with a medium strain hardening exponent and yield strain; Wang et al.'s model has relatively good accuracy for materials with a high strain hardening exponent and yield strain; Carlsson et al.'s model is approximately similar to Suresh et al.'s model in the case of a low strain hardening exponent and yield strain.

1. Introduction

It is well known that residual stresses have significant effects on the mechanical behavior of materials, such as fatigue, fracture, corrosion, wear and friction. Therefore, it is very important to measure residual stresses in engineering structures or components [1]. Up to date, numerous methods for measuring residual stress field [2], such as hole-drilling and layer-removing techniques, curvature measurement, ultrasonic methods, X-ray and neutron diffraction, and recently instrumented indentation, have been developed, among which the instrumented indentation technique (IIT), has attracted intensive interest due to its simplicity, convenience and applicability at various scales.

Both theoretical and experimental investigations [3,4] have shown that residual stresses have significant effect on the indentation load–penetration depth (P–h) curve determined by indentation testing. It has also been found that the hardness increases with the compressive residual stress and decreases with the tensile residual stress. This residual stress dependent hardness variation exhibits remarkable nonlinearity especially for the tensile and compressive cases. Tsui et al. [3] established a bilinear relation between hardness and residual stress. Numerous studies have also indicated that various indentation characteristic parameters such as indentation depth (h), loading curvature (C), contact stiffness (S), and indentation work (W) present nonlinear relationship with residual stresses [5–8]. Based on experimental correlation between the indentation characteristic parameters and residual stress, several models for calculating the residual stresses based on the indentation characteristic parameters have been proposed, among which the models proposed by Suresh and Giannakopoulos [5], Carlsson and Larsson [6,7], Lee and Kwon [9] and Wang et al. [10] are most representative and widely used.

In terms of the research results by Tsui and Bolshakov [3,4], Suresh and Giannakopoulos [5] proposed to use the indentation contact area to characterize the residual stress. Carlsson and Larsson [6,7] established a correlation between the equibiaxial residual stress/strain fields and the contact area/indentation hardness. Lee and Kwon [9] modified the idea of Suresh and defined residual stress σ_R as the differences between the indentation load with and without stresses on the contact area. Wang et al. [10] developed a model for calculating the residual stress from the viewpoint of indentation work during the indentation process. To estimate the residual stresses accurately, the expression should describe the nonlinear effect of residual stresses on indentation response quite well. It can be found that among these models only the method proposed by Suresh and Giannakopoulos [5] clearly described the well-known nonlinear effect of residual stresses from tension to compression [3,4]. Suresh et al. introduced a geometric factor (f_g) in their model to describe this nonlinear relation.

However, many researchers later [11–13] figured out that the nonlinear effect of residual stress was not only determined by the geometry of the indenter, but also dependent on material's mechanical properties, friction and so on. The finite element

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analyses by Huber and Heerens [11] suggested that the nonlinearity in the equibiaxial residual stress state was caused by the hardening behavior of the material. But, Huber et al. did not further give any quantitative expression of the nonlinear effect of residual stress. Chen introduced several indentation parameters to describe the nonlinear relationship and indicated that the ratio of the elastic modulus to the yield strength has great effect on residual stress evaluation. However, in his study the effect of strain hardening on the residual stress was ignored. The finite element simulations by Lee et al. [12] demonstrated that the accuracy of residual stresses evaluation was dependent on material properties, friction coefficient or indenter tip radius, among which the strain hardening behavior was a main factor that led to the nonlinear effect of residual stress on the indentation response.

It can be concluded from the previous studies that the nonlinear effect of residual stress is mainly related to material properties, friction coefficient and indenter tip radius. However, a systematic research work about the nonlinear effect of residual stress has not been reported up to now. For this reason, in the present study the nonlinear effect of the residual stress is tentatively characterized by a correction coefficient (f_g) that is defined as a function of material properties. It is found, by comparing the four representative models for calculating the residual stresses, i.e. Suresh et al.'s model, Lee et al.'s model, Carlsson et al.'s model and Wang et al.'s model, that all these models can be expressed as an expanded form of Suresh et al.'s model when the correction coefficient (f_g) was introduced. The finite element simulations have also been carried out to investigate the effect of material properties especially the strain hardening exponent and yield strain on the accuracy of these four representative models for residual stress calculation.

2. IIT-based representative models for residual stress calculation

2.1. Suresh et al.'s model

Based on the research results by Tsui and Bolshakov [3,4], i.e. residual stress has small effect on material hardness but is very sensitive to the contact area, especially the amount of pile up around the contact area, Suresh and Giannakopoulos [5] proposed to use the indentation contact area to characterize the residual stress.

The model for calculating the residual stress developed by Suresh et al. can be summarized as the principle of stress equivalence, which is schematically illustrated in Fig. 1, where the equibiaxial residual stress σ_R can be separated into a hydrostatic component σ_R^H and a compressive deviator component $-\sigma_R^D$, that is,

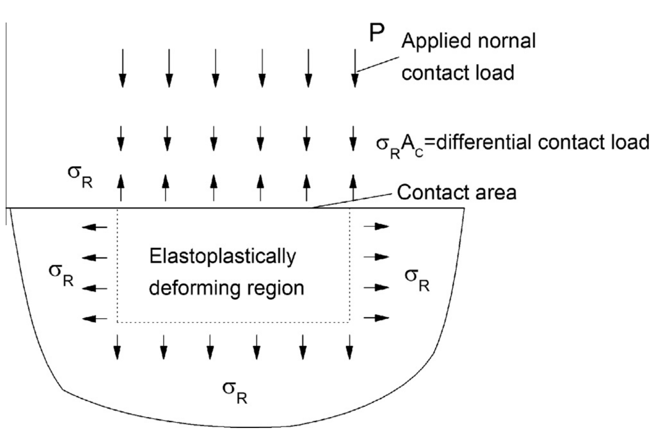

Fig. 1. Schematic of the influence of tensile residual stress on indentation [5].

$$\underbrace{\begin{pmatrix} \sigma_R & 0 & 0 \\ 0 & \sigma_R & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\text{equibiaxial stress}} = \underbrace{\begin{pmatrix} \sigma_R & 0 & 0 \\ 0 & \sigma_R & 0 \\ 0 & 0 & \sigma_R \end{pmatrix}}_{\text{hydrostatic stress}} + \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\sigma_R \end{pmatrix}}_{\text{deviator stress}} \tag{1}$$

As indicated [5], the deviator stress induces a differential indentation force ($P_0 - P$), i.e.,

$$P_0 - P = f_g \cdot \sigma_R A_c \tag{2}$$

where P and P_0 are the maximum loads at the same indentation depth h_{max} with and without residual stress. f_g is defined as the geometric factor that is determined by the geometry of the indenter.

Eq. (2) can also be expressed in the following alternative form,

$$\sigma_R = \frac{1}{f_g} \cdot \frac{P_0 - P}{A_c} \tag{2a}$$

Assuming that the hardness is invariant in residual stress state [5], i.e.,

$$H = \frac{P_0}{A_{c0}} = \frac{P}{A_c} \tag{3}$$

a correlation between the projected contact areas and the residual stress can be established by combining Eqs. (2) and (3),

$$\frac{A_{c0}}{A_c} = 1 + \frac{f \cdot \sigma_R}{H} \tag{4}$$

where A_c and A_{c0} are the true projected contact areas at the same indentation depth h_{max} with and without residual stress. H is the hardness for the material without residual stress. As suggested by Suresh et al., in Eq. (4), $f_g = 1$ corresponds to the tensile residual stress, while $f_g = \sin\theta$ corresponds to the compressive residual stress, where $\theta = \frac{\pi}{2} - \alpha$, with 2α being the included angle of the indenter tip.

2.2. Lee et al.'s model

In terms of the principle of stress equivalence, Lee and Kwon [9] also proposed an indentation-based model for characterizing the residual stress, where the equibiaxial residual stress (σ_R) is separated into the mean and deviatoric components, namely,

$$\underbrace{\begin{pmatrix} \sigma_R & 0 & 0 \\ 0 & \sigma_R & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\text{equibiaxial stress}} = \underbrace{\begin{pmatrix} \frac{2}{3}\sigma_R & 0 & 0 \\ 0 & \frac{2}{3}\sigma_R & 0 \\ 0 & 0 & \frac{2}{3}\sigma_R \end{pmatrix}}_{\text{mean stress}} + \underbrace{\begin{pmatrix} \frac{1}{3}\sigma_R & 0 & 0 \\ 0 & \frac{1}{3}\sigma_R & 0 \\ 0 & 0 & -\frac{2}{3}\sigma_R \end{pmatrix}}_{\text{deviator stress}} \tag{5}$$

From the viewpoint of the shear plasticity, Lee et al. assumed that only the stress component parallel to the indentation axis in the deviatoric components induces plastic deformation. The residual stress (σ_R) can thus be defined as the differences between the indentation loads with and without stresses on the contact area at a given indentation depth, that is,

$$\sigma_R = \frac{3}{2} \frac{P_0 - P}{A_c} \tag{6}$$

Eq. (6) is also further extended to a more general form for estimating the non-equibiaxial residual stress by introducing the stress ratio (κ),

$$\sigma_{R,x} = 3(P_0 - P)/((1 + \kappa)A_c^T) \tag{7}$$

where $\sigma_{R,x}$ is the minor in-plane residual stress component, κ is the ratio of the major in-plane residual stress component $\sigma_{R,y}$ to the minor one $\sigma_{R,x}$ [14].