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Computational homogenization of rubber friction on rough rigid surfaces

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ABSTRACT

The main component of rubber friction is known to be of hysteretic nature, i.e. it is due to viscoelastic energy dissipation taking place in the bulk of the material as a result of the pulsating forces induced by the surface asperities whenever rubber slides on a rough substrate, such as in the case of a car tire on a road surface. This implies that the observed macroscopic friction depends upon the constitutive behavior of the rubber and the characteristics of the rough surface profile. In contrast to analytical models, numerical approaches can fully account for geometric and material non-linearities arising in the rubber behavior, especially at small scales. However, explicit numerical modeling of rough surface features spanning a wide range of significant length scales would result prohibitively expensive, which motivates the need for a computational multiscale framework. As shown by previous related research, fractal surface profiles can be decomposed into a finite number of sinusoidal terms, so that a central ingredient of a multiscale approach becomes the homogenization of rubber friction on a sinusoidal surface. This work proposes a computational homogenization procedure where a macroscale coefficient of friction for rubber is derived from the solution of a microscale boundary-value problem. The latter considers contact of a representative volume element (RVE) with a sinusoidal rigid surface, which is assumed to represent the smallest length scale of a fractal rough surface. The numerical model is developed within the isogeometric framework and features a mortar formulation for the unilateral contact problem in the discretized setting. Numerical aspects related to the choice of the RVE, the setup of the test parameters and the convergence rate of different discretizations are discussed. Physically relevant observations concern the role of the macroscopic applied pressure and sliding velocity on the homogenized friction coefficient. Some comparisons with analytical results as well as dimensional analysis considerations are further reported.

1. Introduction

The frictional contact behavior of elastomers to rough surfaces has a deep impact on several technological applications spanning various fields of engineering, including car tires, seals, wiper blades, conveyor belts, and seismic isolators. For typical combinations of material properties and loading conditions, the rubber components undergo large deformations whereas the contacting surfaces remain nearly undeformed, so that the assumption of rubber contact to a rigid surface is in most cases acceptable and will be maintained throughout this work.

As opposed to many other material combinations, where friction stems primarily from the interaction of the contacting surfaces, rubber friction is mostly generated by energy dissipation inside the material. This conclusion, first reached by Grosch [10] through his pioneering experiments, has been confirmed by numerous investigations and is by now commonly accepted, see e.g. Klüppel and Heinrich [16], Persson [26] and references therein. When rubber slides on a rough substrate, such as in the case of a car tire on a road surface, viscoelastic energy dissipation takes place in the material bulk as a result of the pulsating forces induced by the surface asperities. This implies that the observed macroscopic friction is heavily affected by both the properties of the rubber and the characteristics of the rough contacting surface. A key objective of research is thus to establish a quantitative correlation between the rubber constitutive behavior, the parameters describing the surface roughness, and the resulting macroscopic frictional response. Despite the recent significant progress in this direction, prediction of elastomeric friction in the engineering practice is still a major task and often requires costly large-scale tests as the only reliable option [16].

For the sake of completeness it should be mentioned that the viscoelastic dissipation in the bulk (often referred to in the literature as *hysteretic component* of friction, see e.g. [24]) is only one of the two commonly accepted contributions to rubber friction, the other one being due to the adhesional forces of the interfacial layers. However the latter is only important for dry, clean and

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relatively smooth surfaces, and will therefore be neglected in the present work. Most recently, the existence of a third component due to geometric effects has also been suggested [8].

Extensive experiments have demonstrated that the macroscopic friction coefficient of elastomers depends heavily on various parameters like normal pressure, sliding velocity and surface roughness, see e.g. Le Gal et al. [18], Le Gal and Klüppel [19], Lorenz et al. [21] for recent results. Other important factors such as temperature and lubrication will not be considered in this work, as here dry friction with no thermal effects is addressed. Due to these complex dependencies, the frictional behavior of rubber must be described with appropriate models featuring non-Amontons and non-Coulomb effects. This was done e.g. by Hofstetter et al. [13] within a finite element investigation of tire–road contact. Moreover, many rough surfaces (including typical road surfaces) are known to display self-affine fractal characteristics within a well-defined range of length scales [16,27]. Thus, prediction of the macroscopic rubber friction coefficient on these surfaces, including its observed dependency on pressure and velocity, requires appropriate account for the surface roughness over the whole range of significant length scales.

Analytical theories for the mechanics of rubber contact to self-affine fractal surfaces have been proposed by several authors, most notably by Klüppel and Heinrich [16] and Persson [26]. In the model by Klüppel and Heinrich [16], the hysteretic component of the macroscopic coefficient of friction is obtained by considering a uniaxial viscoelastic element sliding over a rough surface and computing the resulting energy dissipation. The roughness profile is described through its spectral power density, which takes the form of a power law for a self-affine track but may also describe other types of surfaces. The uniaxial element may feature any of the commonly adopted small-deformation viscoelastic models, such as Kelvin–Voigt, Maxwell, or Zener, through the corresponding expressions of the storage and loss moduli of the rubber. As the uniaxial deformation of the viscoelastic element is completely determined by the surface profile, the model implicitly assumes that the surface cavities are completely filled by the rubber, which only holds for sufficiently large normal pressures. This restriction is released by introducing the mean penetration depth of the rubber into the surface, $\langle z_P \rangle$, which is computed through an extension to self-affine surfaces of the classical contact theory by Greenwood and Williamson [9]. In later publications (e.g. [19]), the authors slightly modified the model by substituting $\langle z_P \rangle$ with the thickness of the excited layer, $\langle \delta \rangle \sim \langle z_P \rangle$, thus using $\langle \delta \rangle / \langle z_P \rangle$ as a calibration parameter. The simple model presented by Wriggers and Reinelt [40], based on a static rather than energetic approach, delivers an identical result to that by Klüppel and Heinrich [16] for the special case of a sinusoidal surface.

The model by Persson [26] differs from the theory by Klüppel and Heinrich [16] mainly in two respects: it is fully three-dimensional, and it takes into account to what extent the rubber follows the profile of the rigid substrate at each length scale. This is done via the function $P(\zeta) = A(L/\zeta)/A(L)$, defined as the ratio between the contact area at the length scale L/ζ ($\zeta \geqslant 1$), and the macroscopic contact area $A(L) = A_0$. L is taken as the diameter of the macroscopic contact area, i.e. $A_0 \approx L^2$. This approach is more accurate than the introduction of $\langle z_P \rangle$ or $\langle \delta \rangle$, which serves the same purpose but in an average way. Also this model is based on a small deformation framework, accounts for the characteristics of the surface roughness through its spectral power density, and can incorporate any small-deformation viscoelastic model by accordingly defining the storage and loss moduli of the rubber material.

The analytical models briefly described above, while effectively reproducing the influence of various parameters on the observed frictional response, present clear limitations when a quantitative prediction is sought, mainly due to the assumption of small deformations which is obviously contradicted by the physical behavior. Within a numerical finite element setting, constitutive models suitable for large deformation regimes can be readily incorporated. On the other hand, an explicit numerical modeling of the rough surface features would result prohibitively expensive due to the wide range of significant length scales. Therefore, the need arises for a computational multiscale framework.

Recent years have seen an increasing adoption of computational contact homogenization techniques, whereby macroscopic contact laws are derived from numerical analyses at the microscale which explicitly describe the topography and the constitutive properties of the contacting surfaces [31]. Tworzydlo et al. [38] developed new asperity-based constitutive models of interfaces undergoing normal and frictional contact, through a combination of finite element analysis of the surface asperities and statistical homogenization techniques. Haraldsson and Wriggers [11] applied computational homogenization to obtain frictional contact laws suitable for the interface between soil and concrete. Upon discretisation of the microstructure in the contact area, the plastic deformations of the asperities were simulated numerically using a frictionless contact formulation on the microscale. Orlik et al. [25] used a two-scale computational homogenization technique to derive the effective contact response of a coated cementless hip implant in a human femur based on the microstructural features of the interface layer. Bandeira et al. [3] derived contact interface laws via computational homogenization using a finite element approach for large deformations. Temizer and Wriggers [34] developed a contact homogenization technique to compute the macroscopic coefficient of friction of granular interfaces, i.e. interfaces between an elastic solid and a rigid surface with rigid particles embedded as third bodies. The analysis was extended to granular interfaces with viscoelastic solids by Temizer and Wriggers [35]. Finally, Wriggers and Reinelt [40] formulated a multiscale approach with the objective to derive the macroscopic friction coefficient of rubber on fractal surfaces. The basic idea was the approximation of a fractal surface through the superposition of a discrete set of harmonic functions associated to different length scales. At the smallest length scale friction is neglected. A computation on each scale, where a finite deformation viscoelastic model describes the rubber material, leads through homogenization to a friction law which is locally applied to the next larger scale, where the micro-roughness is defined by the next harmonic function. The procedure is recursively applied until the largest significant scale is reached, leading to the computation of the macroscopic friction law.

In this work, computational contact homogenization is conducted to derive a macroscopic effective coefficient of friction for rubber as a function of sliding velocity and applied pressure. As the multiscale procedure in Wriggers and Reinelt [40] is based on the recursive application of a computational frictional contact model, the final results are quite sensitive to the accuracy of the underlining contact description. A first aim of this research is thus the improvement of the numerical contact model in Wriggers and Reinelt [40]. The finite element solution is now obtained in the framework of isogeometric analysis using non-uniform rational B-splines (NURBS), which improves both the bulk and the surface descriptions. Moreover, the node-to-surface algorithm adopted in Wriggers and Reinelt [40] is substituted with a more accurate and robust mortar contact formulation. For a general background on mortar methods, see e.g. Hild [12], Puso and Laursen [28] and Fischer and Wriggers [7]. A second aim of this work is to present a more detailed discussion of the contact homogenization procedure, thereby analyzing the choice of the test parameters with the aid of appropriate analytical considerations, the representativity of the micromechanical test sample in terms of size and discretization, and the effects of the boundary conditions on the obtained