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# Vibration analysis of micro-scaled sector shaped graphene surrounded by an elastic matrix

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This paper is dedicated to Professor Guo-Wei Wei

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ABSTRACT

Free vibration analysis of micro-scaled annular sector and sector shaped graphene located on an elastic matrix are studied via nonlocal elasticity theory. An eight-node curvilinear element is used for transformation of the governing equation of motion of annular sector graphene from physical region to computational region in conjunctions with the thin plate theory. Elastic matrix is modeled via two-parameters which are Winkler–Pasternak elastic foundations. The discrete singular convolution (DSC) method is employed for numerical solution of resulting nonlocal governing differential equations and related boundary conditions. Then, the effects of nonlocal parameter, mode numbers, sector angles and foundation parameters on the frequency response of micron-scaled annular sector and sector graphene are discussed.



## 1. Introduction

Micro or nano sized mechanical systems are generally modeled as continuous elements such as beams, plates and shells. These components are widely used in biomedical and micro electromechanical purposes and modern industries. Thin films, nano-sheet resonators, biomedical devices, nano electro mechanical applications, micro props, paddle-like resonators, atomic force microscopy, mechanical actuators and nano sensors are examples of these applications. Graphene based structures have been also widely used in micro-electro mechanical systems (MEMS) for high frequency and high sensitive purposes for example molecular gas detectors, solar cells, integrated circuits and nano ribbons due to their ultra mechanical, thermal, optical and electrical properties [1–6]. Mechanical properties of the graphene sheets are widely investigated by researchers [7–12] in the past ten years.

It is known that the analysis based on the classical elasticity theory does not take into consider the internal length scale effect of nanostructure. In order to introduce the size effect to the governing equations, material length scale parameters must be taken into account. By this time, a few different approaches such as atomistic modeling, atomistic-continuum coupled methods and continuum modeling have been used for modeling and analysis of these nano-scaled systems. Atomistic simulation model or hybrid atomistic-continuum model are computationally expensive. So, some higher-order continuum theories have been proposed by this time. In the early of 1970s, nonlocal elasticity theory is proposed by Eringen [13] for modeling of the length-scale problems in continuum mechanics. This theory is widely used by researchers for modeling of nano scaled structures [14–22]. Zhang et al. [23,24] investigated the vibration of carbon nanotubes via nonlocal elasticity. Radial buckling pressure of a simply supported multi-walled carbon nanotube is presented by Xie et al. [25]. Free vibration of orthotropic arbitrary straight-sided quadrilateral nanoplates is analyzed via nonlocal elasticity theory [26,27]. Aksencer and Aydogdu [28] and Aydogdu [29] applied the nonlocal elasticity to the vibration of nano-scaled plates. Numerical and analytical methods are generally used for solution of nonlocal plate problem [30–37]. Vibration and bending analyses of Micro-scaled beams and carbon nanotubes are also modeled by nonlocal elasticity [38–44]. Vibration, buckling and bending analyses of protein microtubules and carbon nanotubes are detailed investigated by present authors [45–50] using higher-order elasticity theory. Micro or nano-scaled plates are generally used in MEMS purposes devices in many applications such as ultra thin films, nano sheet resonators, graphene sheets. There have been a number of studies in recent past ten years dealing with the mechanical properties of nano-scaled beams and plates via nonlocal continuum mechanics under different mechanical and thermo elastic conditions. In the

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literature; rectangular and circular nano or micro plates have been investigated via nonlocal elasticity. The effects of elastic matrix on frequency had been investigated just for rectangular and circular micro plates or graphene in the past. In the present study, however, free vibration analysis of micro-scaled annular sector and sector graphene resting on an elastic matrix is firstly investigated using the geometric transformation based on the nonlocal continuum theory in conjunction with the discrete singular convolution method.

This paper is organized as follows. Section 2 is devoted to a brief description of the DSC method. Section 3 gives some formulations for the geometric mapping from the physical domain to computational domain. Nonlocal elasticity theory is presented in Section 4. Section 5 presents the application of the numerical schemes to the solution of nonlocal equation of micro-scaled annular sector graphene on elastic matrix. Some numerical examples are presented in Section 6. Conclusions are presented in Section 7.

## 2. Discrete singular convolution (DSC)

Finite difference and finite element methods had been widely used in engineering problems over fifty years. Singular convolutions are the special branch of mathematical transformation area. The generally transforms are known as Radon, Abel and Hilbert. The method of discrete singular convolution has recently been proposed for engineering and mathematical physics problems by Wei [51] in 1999 via theory of distributions. After this, Wei [52] first introduced this method for solving mechanical problems. By this time, a variety of structural mechanics problems have been analyzed using the method of DSC [42–58] in successfully. In the present paper, details of the DSC method are not given; interested readers may refer to the works of [59–64]. Consider a distribution, $T$ and $\eta(t)$ as an element of the space of test function. A singular convolution can be defined by [51]

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx \tag{1}$$

where $T(t-x)$ is a singular kernel. For example, singular kernels of delta type [52]

$$T(x) = \delta^{(n)}(x); \quad (\mathrm{n} = 0, 1, 2, \ldots,) \tag{2}$$

Kernel $T(x) = \delta(x)$ is important for interpolation of surfaces and curves, and $T(x) = \delta^{(n)}(x)$ for $n > 1$ are essential for numerically solving differential equations. With a sufficiently smooth approximation, it is more effective to consider a discrete singular convolution [53]

$$F_\alpha(t) = \sum_k T_\alpha(t - x_k) f(x_k) \tag{3}$$

where $F_\alpha$ (t) is an approximation to $F(t)$ and $\{x_k\}$ is an appropriate set of discrete points on which the DSC is well defined [32–35]. Note that, the original test function $\eta(x)$ has been replaced by $f(x)$. This new discrete expression is suitable for computer realization. The mathematical property or requirement of $f(x)$ is determined by the approximate kernel $T_\alpha$. Recently, the use of some new kernels and regularizer such as delta regularized was proposed to solve applied mechanics problem. The researchers are generally used the regularized delta Shannon kernel [30–39]. The Shannon's kernel is regularized as [54]

$$\delta_{\Delta,\sigma}(x - x_k) = \frac{\sin[(\pi/\Delta)(x - x_k)]}{(\pi/\Delta)(x - x_k)} \exp\left[-\frac{(x - x_k)^2}{2\sigma^2}\right]; \quad \sigma > 0 \tag{4}$$

where $\Delta$ is the grid spacing. It is also known that the truncation error is very small due to the use of the Gaussian regularizer, the above formulation given by Eq. (4) is practical, and has an essentially compact support for numerical interpolation. Equation (4) can also be used to provide discrete approximations to the singular convolution kernels of the delta type [54]

$$f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta_\Delta(x - x_k) f(x_k) \tag{5}$$

where $\delta_\Delta(x - x_k) = \Delta\delta_\alpha(x - x_k)$ and superscript $(n)$ denotes the n*th*-order derivative. The $2M + 1$ is the computational bandwidth which is centered around $x$, and is usually smaller than the whole computational domain. In the DSC method, the function $f(x)$ and its derivatives with respect to the $x$ coordinate at a grid point $x_i$ are approximated by a linear sum of discrete values $f(x_k)$ in a narrow bandwidth $[x - x_M, x + x_M]$. This can be expressed as [55]

$$\left.\frac{d^n f(x)}{dx^n}\right|_{x=x_i} = f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta^{(n)}_{\Delta,\sigma}(x_i - x_k) f(x_k); \quad (\mathrm{n} = 0, 1, 2, \ldots,) \tag{6}$$

where superscript $n$ denotes the $n$th-order derivative with respect to $x$. The $x_k$ is a set of discrete sampling points centered around the point $x$, $\sigma$ is a regularization parameter, $\Delta$ is the grid spacing, and $2M + 1$ is the computational bandwidth which is usually smaller than the size of the computational domain [53–56]. For example, the second order derivative at $x = x_i$ of the DSC kernels for directly given [52]

$$\delta^{(2)}_{\Delta,\sigma}(x - x_j) = \frac{d^2}{dx^2}[\delta_{\Delta,\sigma}(x - x_j)]|_{x=x_i} \tag{7}$$

The discretized forms of Eq. (7) can then be expressed as

$$f^{(2)}(x) = \left.\frac{d^2 f}{dx^2}\right|_{x=x_i} \approx \sum_{k=-M}^{M} \delta^{(2)}_{\Delta,\sigma}(k\Delta x_N) f_{i+kj} \tag{8}$$

## 3. Geometric mapping

By using the transformation rule, a nonrectangular physical domain can be easily transformed into a normalized computational domain via geometric mapping. This technique has been widely used in the finite elements and differential quadrature methods by this time. In order to transform from physical domain to computational domain, let consider an eight-node curvilinear quadrilateral domain as shown in Fig. 1a. Thus, the following equations are used for the coordinate transformation [65]

$$x = \sum_{i=1}^{8} \Psi_i(\xi, \eta) x_i \tag{9}$$

$$y = \sum_{i=1}^{8} \Psi_i(\xi, \eta) y_i \tag{10}$$

Hence, first-order, and second order derivatives of a function are given via chain rule

$$\begin{Bmatrix} u_x \\ u_y \end{Bmatrix} = [J_{11}]^{-1} \begin{Bmatrix} u_\xi \\ u_\eta \end{Bmatrix} \tag{11}$$

$$\begin{Bmatrix} u_{xx} \\ u_{yy} \\ 2u_{xy} \end{Bmatrix} = [J_{22}]^{-1} \begin{Bmatrix} u_{\xi\xi} \\ u_{\eta\eta} \\ u_{\xi\eta} \end{Bmatrix} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1} \begin{Bmatrix} u_\xi \\ u_\eta \end{Bmatrix} \tag{12}$$

where $\xi_i$ and $\eta_i$ are the coordinates of Node $i$ in the $\xi$–$\eta$ plane, and $J_{ij}$ are the elements of the Jacobian matrix. These are expressed as follows [65,66];

$$[J_{11}] = \begin{bmatrix} x_\xi & y_\xi \\ x_\eta & y_\eta \end{bmatrix} \tag{13}$$