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Willis elastodynamic homogenization theory revisited for periodic media

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ABSTRACT

The theory of elastodynamic homogenization initiated by J.R. Willis is revisited for periodically inhomogeneous media through a careful scrutiny of the main aspects of that theory in the 3D continuum context and by applying it to the thorough treatment of a simple 1D discrete periodic system. The Bloch theorem appears to be central to appropriately defining and interpreting effective fields. Based on some physical arguments, three necessary conditions are derived for the transition from the microscopic description to the macroscopic description of periodic media. The parameters involved in the Willis effective constitutive relation are expressed in terms of two localization tensors and specified with the help of the corresponding Green function in the spirit of micromechanics. These results are illustrated and discussed for the 1D discrete periodic system considered. In particular, inspired by Brillouin's study, the dependency of the effective constitutive parameters on the frequency is physically interpreted in terms of oscillation modes of the underlying microstructure.

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1. Introduction

The beginnings of the elastodynamic homogenization theory of J.R. Willis can be traced back to the relevant papers he published during the first half of the 1980s (Willis, 1980a,b, 1981, 1985). The main body of this theory was, in a rather complete manner, presented more than 10 years later in a chapter of a book edited after a course dedicated to continuum micromechanics (Willis, 1997). Recently, increasing interest in acoustic metamaterials and cloaking (see, e.g., papers by Chen and Chan, 2010; Lee et al., 2012; Liu et al., 2000, 2012; Milton et al., 2006; Norris, 2008; Norris and Shuvalov, 2011; Simovski, 2007) has, in particular, given an impetus to the development and application of the elastodynamic homogenization theory of Willis (Milton and Willis, 2007, 2010; Nemat-Nasser and Srivastava, 2011, 2013; Nemat-Nasser et al., 2011; Norris et al., 2012; Shuvalov et al., 2011; Srivastava and Nemat-Nasser, 2011; Willis, 2009, 2011, 2012).

The elastodynamic homogenization theory of Willis exhibits the following salient features: (i) in the microscopic-to-macroscopic upscaling process, no approximation hypotheses are made, so that, in this sense, the resulting theory can be considered as exact; (ii) the effects of material microscopic inhomogeneities are, after homogenization, all incorporated only in the resulting non-classical effective constitutive law, so that the macroscopic (or effective) motion equation takes the same classical form as the one at the microscopic level; (iii) for a composite formed of elastic phases whose constitutive laws are local in time and space, the effective constitutive law obtained by homogenization becomes generally nonlocal both in time and space; (iv) the effective mass density is, in general, no longer a scalar but a second-order tensor quantity; (v) a non-classical coupling between the effective

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stress tensor and the effective velocity, and another one between the effective momentum and the effective strain tensor, occur generally in the effective constitutive law; (vi) the parameters involved in the effective constitutive law are non-unique but can be rendered unique by prescribing, for example, an additional eigenstrain field. Note that the features (iii)–(vi) make that the effective constitutive law derived in the homogenized elastodynamic theory of Willis is very different from the constitutive law involved in the classical elastodynamic theory and that its explicit determination in terms of the phase properties is a quite tough task and in general necessitates using a numerical method.

The present work consists in revisiting the elastodynamic homogenization theory of Willis for periodic composites so as to reach the following threefold objective. First, it aims to derive, on the basis of some physically sound arguments, a few necessary conditions for the application of that theory to be physically meaningful. Second, it has the purpose of expressing the effective constitutive parameters of the effective constitutive law in terms of some appropriate localization tensors in the spirit of micromechanics, so that a general numerical method, such as the finite element method, can be directly used to numerically compute said parameters. Thirdly, it aims to gain physical insights into the general theory by applying it to thoroughly and analytically study a simple one-dimensional (1D) periodic discrete system. By achieving these three objectives, the present work contributes not only to getting a better understanding of but also developing the elastodynamic homogenization theory initiated by Willis.

The next sections of this paper are structured and summarized as follows.

In Section 2, which is the main part of this paper, the elastodynamic homogenization theory of Willis is carefully reformulated mathematically and examined physically for periodic composites. After providing some geometrical preliminaries and recalling the local classical elastodynamic equations for periodic media, Bloch theorem is shown to play a central role in solving the corresponding local motion equation and in properly defining and interpreting the effective (or macroscopic) fields. Three necessary conditions are then proposed for the elastodynamic homogenization theory of Willis applied to a periodic composite to lead to a physically meaningful effective behavior. The first necessary condition corresponds to the requirement that the microscopic virtual work be equal to the effective (or macroscopic) virtual work, which is reminiscent of the well-known Hill–Mandel lemma in micromechanics. The second necessary condition concerns wavenumbers and demands that the effective (or macroscopic) fields capture the long-wavelength parts of the relevant microscopic fields. The third necessary condition is relative to frequencies and comes from the requirement that the effective elastodynamic behavior of a composite be a good approximation of its microscopic one. In the last part of Section 2, two localization tensors are first introduced in the spirit of micromechanics and then explicitly expressed in terms of the relevant Green function for a given pair of frequency and wavenumber. With the help of the expressions for the localization tensors, the frequency- and wavenumber-dependent parameters characterizing the effective elastodynamic constitutive law are finally specified in terms of the Green function. The obtained effective elastodynamic behavior is then reinterpreted physically in light of the established homogenizability conditions.

In Section 3, inspired by the work of Brillouin (1953), a simple 1D periodic discrete system is analytically and exhaustively studied to illustrate and discuss the main results of Section 2. In particular, the effective constitutive parameters are analytically and exactly obtained, and the effective impedance and dispersion relation are derived and illustrated. The homogenizability conditions proposed in Section 2 are discussed.

Finally, in Section 4, a few concluding remarks are drawn and some open problems are mentioned.

2. Elements of an elastodynamic homogenization theory for periodic media

We start by presenting some useful geometrical tools to the study of periodic media and the basic motion and constitutive equations. The motion equation is subsequently simplified and restricted to one arbitrary unit cell thanks to Bloch–Floquet theorem. The Bloch-wave-expansion leads to a definition of the effective fields and the corresponding effective behavior. We then thoroughly discuss the consistency of the Willis approach according to three criteria hereafter called "homogenizability conditions". Such a discussion is possible even before having any expression of the effective constitutive equations. The derivation of said expression is presented last.

2.1. Problem set-up

2.1.1. Geometrical considerations

Let Ω be a periodic medium. We liken Ω to a 3D point space with an underlying vector space called $\mathcal{E}$ and a periodicity lattice $\mathcal{R}$. Vectors of $\mathcal{E}$ can be identified with points of Ω through the choice of an arbitrary origin. The lattice $\mathcal{R}$ is defined as

$$\mathcal{R} = \{\boldsymbol{r} \in \mathcal{E} | \boldsymbol{r} = a_1\boldsymbol{b}_1 + a_2\boldsymbol{b}_2 + a_3\boldsymbol{b}_3,\ a_i \in \mathbb{Z}\}$$

where $\mathbb{Z}$ stands for the set of integers and the vectors $(\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3)$ form a basis for $\mathcal{E}$. A unit cell T for Ω can then be specified as

$$T = \{\boldsymbol{r} \in \mathcal{E} | \boldsymbol{r} = r_1\boldsymbol{b}_1 + r_2\boldsymbol{b}_2 + r_3\boldsymbol{b}_3,\ r_i \in [0, 1[\}.$$

Note that neither the choice of $(\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3)$ nor that of T is unique. In Fig. 1, a two-dimensional (2D) lattice $\mathcal{R}$ is illustrated.

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