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Optimal scaling laws for ductile fracture derived from strain-gradient microplasticity

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A B S T R A C T

We perform an optimal-scaling analysis of ductile fracture in metals. We specifically consider the deformation up to failure of a slab of finite thickness subject to monotonically increasing normal opening displacements on its surfaces. We show that ductile fracture emerges as the net outcome of two competing effects: the sublinear growth characteristic of the hardening of metals and strain-gradient plasticity. We also put forth physical arguments that identify the intrinsic length of strain-gradient plasticity and the critical opening displacement for fracture. We show that, when J_c is renormalized in a manner suggested by the optimal scaling laws, the experimental data tends to cluster—with allowances made for experimental scatter—within bounds dependent on the hardening exponent but otherwise material independent.

1. Introduction

Ductile fracture is the process whereby a material separates across a failure surface through mechanisms, such as void nucleation, growth and coalescence, that entail large amounts of plastic work (cf., e.g., McClintock and Argon, 1966; Rice, 1968; Hutchison, 1979; Kanninen and Popelar, 1985 for accounts of the subject; see also Fig. 1 for an illustration of the mechanism of failure under consideration). Thus, unlike brittle fracture, where the behavior of the material is ostensibly elastic up to fracture, in ductile fracture plastic dissipation accounts for a significant—or even dominant—part of the energy budget. Such extensive plastic deformation notwithstanding, ductile fracture remains quintessentially a fracture process, in the sense that failure takes place by separation across a plane or surface and entails a well-characterized amount of energy *per unit area*, or *specific fracture energy*, to operate. Ductile fracture constitutes a design-limiting mode of failure in many engineering applications, such as pressure vessels and piping, that call for the use of ductile metals such as mild steels. Experimentally, ductile fracture is easily identified fractographically, as the crack surfaces exhibit a characteristic dimpling—the dimples being vestiges of voids—that is in stark contrast to the sharp specular cracks that result from brittle fracture. Furthermore, the measured specific fracture energies attendant to ductile fracture, e.g., from Charpy tests or from J-testing (Kanninen and Popelar, 1985), are much larger than those of brittle solids and exhibit a characteristic temperature dependence that includes a brittle-to-ductile transition at a critical temperature.

Owing to its engineering importance, ductile fracture has been the focus of extensive study since the 60s and one of the main driving forces in the development of nonlinear fracture mechanics. A number of micromechanical and computational models have been put forth (cf., e.g., Ritchie et al., 1973; Rice and Thomson, 1974; Needleman, 1982; Tvergaard, 1982, 1990,

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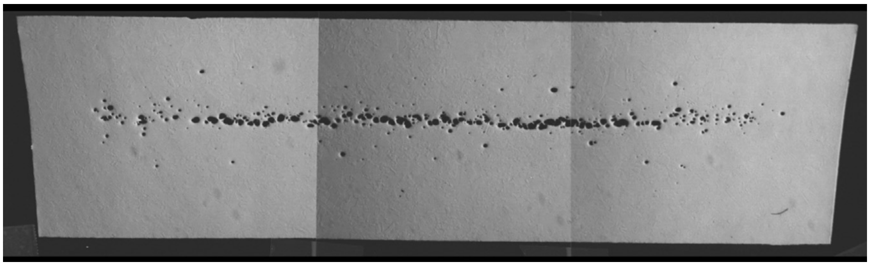

Fig. 1. Photomicrograph of a copper disk tested in a gas-gun experiment showing the formation of voids and their coalescence into a fracture plane (Heller, 2002).

1992, 2007; Becker et al., 1988, 1989; Koplik and Needleman, 1988; Xu and Needleman, 1991; Needleman and Tvergaard, 1991; Tvergaard and Hutchinson, 2002; Pardoen and Hutchinson, 2003; Tvergaard and Niordson, 2004; Xue et al., 2010; Nielsen and Tvergaard, 2011), including consideration of nonlocal effects (cf., e.g., Tvergaard and Needleman, 1997; Borg et al., 2006; Niordson, 2008; Nielsen et al., 2012) and scaling (Needleman et al., 2012), that account for the observational evidence and relate macroscopic properties to material structure and behavior at the microscale. These insightful advances notwithstanding, the connection between micromechanical properties and ductile fracture, and specifically any scaling laws thereof, has defied rigorous analytical treatment and characterization, to a large extent for want of effective and workable tools of analysis. A first essential difficulty stems from the dissipative, path-dependent and hysteretic nature of plasticity. However, under the ostensibly proportional loading conditions that may be expected to prevail during fracture under monotonically increasing loads, *deformation theory of plasticity* (cf., e.g., Martin, 1975) may be expected to supply a good approximation. Deformation theory reduces the plasticity problem to a sequence of minimization problems, the functional to be minimized accounting for both energy and dissipation. This variational structure in turn opens the way to the application of the tools of the modern calculus of variations. Of special interest here is the identification of *optimal scaling laws* relating the macroscopic behavior to micromechanical and loading parameters. Such optimal scaling laws are established but producing upper and lower bounds of a power-law type with matching exponents for all parameters in both bounds. Optimal scaling methods were pioneered by Kohn and Müller (1992) as part of their seminal work on branched structures in martensite, and have been since successfully applied to a number of related problems, including shape-memory alloys, micromagnetics, crystal plasticity, and others (Kohn and Müller, 1992, 1994; Choksi et al., 1999; Conti, 2000; Conti and Ortiz, 2005).

Fokoua et al. (in press) have recently performed an optimal-scaling analysis of ductile fracture in metals. They specifically consider the deformation, ultimately leading to fracture, of a slab of finite thickness subject to monotonically increasing normal opening displacements on its surfaces. Fokoua et al. (in press) posit two competing constitutive properties, namely, *sublinear energy growth* and *strain-gradient hardening*. Sublinear growth (the energy of linear elasticity exhibits quadratic growth, by way of comparison) is a reflection of the work-hardening characteristics of conventional metallic specimens and gives rise to well-known geometric instabilities such as the necking of bars, sheet necking, strain localization and others (cf., e.g., McClintock and Argon, 1966). Strain-gradient hardening (Fleck and Hutchinson, 1993, 1997, 2001; Fleck et al., 1994) has been extensively investigated and demonstrated by means of torsion tests in wires (Fleck et al., 1994), nanoindentation (Nix and Gao, 1998; Xue et al., 2000; Huang et al., 2000), and by other means. It results in deviations from volume scaling, i.e., in *nonlocal behavior* and *size dependency*, in sufficiently small material samples.

Fokoua et al. (in press) show that ductile fracture indeed emerges as the net outcome of these two competing effects: whereas the sublinear growth of the local energy promotes localization of deformation to failure planes, strain-gradient plasticity stabilizes this process of localization in its advanced stages, thus resulting in a well-defined specific fracture energy. Specifically, they show that ductile fracture requires a well-defined energy per unit area that can be bounded above optimally by a void-sheet construction. This specific fracture energy bears a power-law relation to the prescribed opening displacement. This power-law relation may be regarded as an effective cohesive potential, thus indicating that ductile fracture is cohesive in nature. In particular, fracture processes involving distributed—possibly fractal—damage are ruled out by the analysis.

In this paper we endeavor to make connection between the scaling laws of Fokoua et al. (in press) and engineering fracture properties such as J_c, the plane-strain value of the J-integral at crack-growth initiation (cf., e.g., Kanninen and Popelar, 1985). However, in order to compare with actual test data, the analysis of Fokoua et al. (in press) needs to be refined and extended. Thus, the main focus of optimal scaling analysis, including the analysis of Fokoua et al. (in press), is the determination of the optimal scaling exponents, which are a rigorous outcome of the analysis, and scant effort is often put into deriving tight constants. However, in performing quantitative comparisons with experiment the accuracy and tightness of the constants becomes a critical concern. Therefore, we proceed to explicate and improve the constants supplied by Fokoua et al. (in press), both for the upper and the lower bounds.

In addition, within the present framework the evaluation of J_c for specific materials requires knowledge of their intrinsic length ℓ of strain-gradient plasticity and critical opening displacement δ_c. The direct identification of ℓ requires the execution of specially designed experiments such as torsion of wires and nanoindentation (cf. Fleck et al., 1994; Begley and