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Analysis of stress distributions under lightweight wheeled vehicles

C. Senatore *, K. Iagnemma

Laboratory for Manufacturing and Productivity, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

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Abstract

In recent years, the need for reliable modeling tools for lightweight robotic systems deployed on various terrains has spurred research efforts into development of vehicle terrain interaction (VTI) models. This paper presents an analysis of rigid wheels – dry sand interaction and compares experimental results with predictions from established terramechanics theory. A novel experimental setup, based on sensing elements placed on the wheel surface, allows inference of normal and tangential stress at the wheel-terrain interface. A particle image velocimetry (PIV) analysis is used to study the soil kinematics under the wheel. The analysis of stress profiles shows that stress patterns under lightweight vehicle wheels conform reasonably well to established terramechanics theory developed for heavy vehicles. For the wheel under investigation, the stress distribution had minor variation along wheel width for low slip conditions. The wheel model proposed by Wong and Reece was analyzed in light of the stress and soil kinematics measurements available. It was found that, by appropriately characterizing the model coefficients c_1 and c_2, and understanding the physical meaning of the shear modulus k_x, it is possible to obtain torque, drawbar force, and sinkage predictions within 11% (full scale error) of experimental data.

Keywords: Wheel model; Shear strength; Stress sensor; Granular particle image velocimetry; Terrain shearing failure; Wheel dynamics; Off-road vehicle performance

1. Introduction

In recent years, the analysis of lightweight robotic system mobility has raised many questions regarding whether classical terramechanics theory for wheeled vehicles is accurately predictive for reduced scale vehicles [8,18,4,12]. Lacking a standardized classification, in this paper we arbitrarily define lightweight vehicles as having average ground pressure below 20 kPa. Many space rovers and robotic ground vehicles fall within this classification.

Basing his analysis on fundamental concepts of soil mechanics, Bekker [2] introduced a theory to predict mobility of wheeled and tracked vehicles in off-road scenarios. Bekker proposed a set of semi-empirical equations to predict different mobility aspects, such as compaction resistance, traction, sinkage, and driving torque. Bekker himself noted that terramechanics theory “become less accurate for wheels smaller than 20 in. [. . .] and for wheel loads below about 10 lbs”. Carrier [8], while studying the trafficability of lunar micro-rovers, concluded that classical Bekker equations lead to an underestimation of small rover tractive performance. Richter et al. [18] investigated the performance of wheels with diameter ranging between 150 mm and 250 mm and vertical loads ranging from 10 N up to 120 N, and concluded that classical Bekker model needs corrections in order to accurately predict performance. Meirion-Griffith and Spenko [12] used small wheels as penetration plates, and noted that Bekker’s pressure–sinkage Eq. (21) is affected by wheel curvature. Griffith and Spenko proposed a modified Bekker pressure–sinkage equation to account for small wheels’ curvature.

The theory for off road rigid wheel mobility evaluation developed by Bekker was further refined by Wong and Reece [25,26]. Wong and Reece did not simply apply

* Corresponding author.
E-mail addresses: senator@mit.edu (C. Senatore), kdi@mit.edu (K. Iagnemma).

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correction factors to Bekker equations, but rather expanded the Bekker methodology to calculate wheel performance through the prediction of stress distributions at the wheel-terrain interface. The model proposed by Wong and Reece (here referred to as the WR model) has the merit of deriving all wheel performance metrics (i.e., drawbar force, torque, and sinkage) from the calculated stress distributions at the interface. On the other hand, Bekker's original approach was based on a series of ad hoc formulations intended to model each single aspect of vehicle mobility independently.

Ishigami et al. [9] showed that a WR-based model could reasonably replicate single wheel experiments (though only positive slip was investigated). However, in [9] it is not discussed how soil parameters were calculated, and therefore it is reasonable to assume that some soil parameters were tuned to match the experimental observations. Ding et al. [4], noting a significant discrepancy between measured and predicted sinkage, proposed a modified WR model where the sinkage exponent (see A) is modified according to slip. Based on the authors' own experience, tuning of WR model parameters is inevitably required to achieve accurate model predictions across a broad range of loading and slip conditions.

This brief overview of the most significant work on lightweight vehicle mobility modeling shows that previous studies have either proposed modifications of the Bekker–Wong–Reece models (typically by introducing additional parameters) or they have arbitrarily tuned some parameters to improve correlation with experimental data. In either case, the inherent reasons for poor model performance were not investigated.

To overcome these issues, in this paper we describe a detailed analysis of stress distributions under small-sized rigid wheels operating on cohesionless soil in order to understand if, where, and how WR models fail. (Note that the original Bekker model is not discussed.) A custom force sensing array located at the wheel-terrain interface is used to measure stresses at the wheel interface. The force sensors are strain gage-based flexural elements with interchangeable interface surfaces that are designed for integration with wheels or other running gear. The sensors allow explicit measurement of normal and shear forces (and, therefore, estimation of normal and shear stresses) at numerous discrete points along the wheel-soil interface. Similar experimental methodologies were employed by Hegedus [6], Sela [19], Onafeko and Reece [17], Krick [11], and Shamay [22]. The key difference is that in [6,19,17,11,22] the average wheel ground pressure was approximately 100 kPa, while in this paper the wheel average ground pressure is on the order of 10 kPa. (The average ground pressure is evaluated as the nominal wheel load distributed over a flat wheel section spanning 30°.) Oida et al. [16] instrumented a flexible tire with a sensor, based on Krick's design [11], and they measured normal, tangential, and lateral stress at the tire-sand interface (however the vertical load and tire dimensions are unknown). Nagatani et al. [15] have used stock button-type force transducers to measure normal stress at wheel-terrain interface. Although the average ground pressure was comparable to what is studied here, the setup proposed in [15] was only able to measure normal load.

Another experimental methodology employed in this work relies on imaging of the wheel-soil interface and the use of particle image velocimetry (PIV) to measure micro-scale terrain displacement. This methodology, although confined to a plane strain case, allows measurement of the soil displacement field under the wheel. Though, this method does not explicitly permit calculation of the velocities of individual soil particles, it does allow estimation of a regularly-spaced velocity field in the soil. While such visualization techniques have been widely employed in the field of experimental fluid mechanics, their application to the study of soils is a relatively new development [23,13,14].

Measurements of stress distributions and the soil velocity field are complemented by an in-depth comparison with WR model predictions. The model relies on a set of 6 terrain parameters and 3 wheel-terrain interaction coefficients, presented in Table 1. This work identifies the shear modulus, k_x, and the coefficients for determining the relative position of the maximum radial stress, c_1 and c_2, as the principal factors that often lead to poor performance of WR model predictions.

Here, we have confined our study to wheel operation on dry sand. The sand utilized in this paper has been fully characterized via a series of direct shear tests (ASTM D3080) and penetration tests. Direct shear tests were performed to estimate shearing parameters such as cohesion, c, angle of internal friction, ϕ, and shear modulus k_x. Penetration tests, although not standard tests, were performed to evaluate the "Bekker parameters" n, k_c, and k_ϕ, which are necessary for characterization of the pressure–sinkage behavior of the soil. The key questions that this paper addresses are:

Q1 Are the stress distributions that form under lightweight wheels similar in nature to those that form under heavy weight vehicles?

Q2 Is the WR wheel model capable of accurately modeling lightweight vehicle mobility?

Table 1
Wong and Reece wheel model terrain parameters and coefficients.

Symbol	Units	Description
n	n/a	Sinkage exponent
k_c	kN/m^{n+1}	Pressure–sinkage coefficient
k_ϕ	kN/m^{n+2}	Pressure–sinkage coefficient
c	Pa	Cohesion
ϕ	°	Angle of internal friction
k_x	m	Shear deformation modulus
c_1, c_2	n/a	Coefficients for determining the relative position of maximum radial stress
θ_r	°	Exit angle