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Thermomechanics of shells undergoing phase transition

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ABSTRACT

The resultant, two-dimensional thermomechanics of shells undergoing diffusionless, displacive phase transitions of martensitic type of the shell material is developed. In particular, we extend the resultant surface entropy inequality by introducing two temperature fields on the shell base surface: the referential mean temperature and its deviation, with corresponding dual fields: the referential entropy and its deviation. Additionally, several extra surface fields related to the deviation fields are introduced to assure that the resultant surface entropy inequality be direct implication of the entropy inequality of continuum thermomechanics. The corresponding constitutive equations for thermoelastic and thermoviscoelastic shells of differential type are worked out. Within this formulation of shell thermomechanics, we also derive the thermodynamic continuity condition along the curvilinear phase interface and propose the kinetic equation allowing one to determine position and quasistatic motion of the interface relative to the base surface. The theoretical model is illustrated by two axisymmetric numerical examples of stretching and bending of the circular plate undergoing phase transition within the range of small deformations.

1. Introduction

Phase transition (PT) phenomenon in continuous media originally described by Gibbs (1928) was developed in a number of papers summarised in several recent books for example by Grinfeld (1991), Gurtin (1993, 2000), Romano (1993), Sun (2002), Bhattacharya (2003), Fischer (2004), Abeyaratne and Knowles (2006), Lagoudas (2008), and Berezovski et al. (2008). In this approach one assumes existence of the sharp phase interface being a sufficiently regular surface dividing different material phases. The position and motion of the phase interface itself is among the most discussed issues in the field. In the literature many model one-dimensional (1D) problems were analysed theoretically, numerically and experimentally which adequately described behaviour of bars, rods, and beams made of martensitic materials.

Experiments on shape memory alloys and other materials undergoing PT are often performed with thin-walled samples such as thin strips, rectangular plates or thin tubes, see Li and Sun (2002), He and Sun (2009a,b, 2010a,b), and Sun (2002) among others. One would expect that two-dimensional (2D) thermomechanics describing the behaviour of thin-walled structural elements made of materials undergoing PT which is based on the theory of shells was developed long ago. But this is not the case. To our best knowledge a simple 2D mechanical model of PT in thin films was proposed by Bhattacharya and James (1999), James and Rizzoni (2000), and Shu (2000), see also Bhattacharya (2003) and Miyazaki et al. (2009).

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The model consists of the Cosserat membrane with one director, but without taking into account bending rigidity of the membrane. Alternative simple models of PT in biomembranes were proposed by Boulbitch (1999), Agrawal and Steigmann (2008), and Elliott and Stinner (2010).

The non-linear equilibrium conditions of elastic shells undergoing PT of martensitic type were formulated by Eremeyev and Pietraszkiewicz (2004) and Pietraszkiewicz et al. (2007) within the dynamically exact and kinematically unique theory of shells developed by Libai and Simmonds (1983, 1998) and Chróścielewski et al. (2004). This version of the non-linear theory of shells has the structure of the classical Cosserat surface with the translation vector $\boldsymbol{u}$ and rotation tensor $\boldsymbol{Q}$ fields as the only independent variables. By analogy to the 3D case, the two-phase shell was regarded in Eremeyev and Pietraszkiewicz (2004) and Pietraszkiewicz et al. (2007) as the Cosserat surface consisting of two material phases divided by a sufficiently smooth surface curve. Existence of such a curve was confirmed by several experiments on thin-walled samples. For such a general shell model the first 2D thermomechanical model of PT has recently been worked out by Eremeyev and Pietraszkiewicz (2009).

In this paper we develop the general non-linear thermomechanics of the resultant Cosserat-type shells undergoing diffusionless (displacive) phase transitions of martensitic type. In particular, we discuss the thermodynamic condition allowing one to determine position and quasistatic motion of the phase interface on the deformed shell base surface. Here we use extended thermodynamics of shells based on the introduction of two temperature fields. The theoretical model is illustrated by example of stretching and bending of the circular plate undergoing phase transition in the case of small deformations.

2. Kinematics

In the undeformed placement the shell-like body is represented by the base surface M described by the position vector $\boldsymbol{x}(\theta^\alpha)$, and orientation of M is defined by the unit normal vector $\boldsymbol{\eta}(\theta^\alpha)$, with $\{\theta^\alpha\}$, $\alpha=1,2$, the surface curvilinear coordinates.

Within the dynamically exact and kinematically unique theory of shells summarised in Libai and Simmonds (1998), Chróścielewski et al. (2004), Eremeyev and Zubov (2008), in the deformed placement the shell is represented by the position vector $\boldsymbol{y}=\chi(\boldsymbol{x})$ of the deformed material base surface $N=\chi(M)$ with attached three directors $(\boldsymbol{d}_\alpha,\boldsymbol{d})$ such that

$$\boldsymbol{y}=\boldsymbol{x}+\boldsymbol{u}, \quad \boldsymbol{d}_\alpha=\boldsymbol{Q}\boldsymbol{x}_{,\alpha}, \quad \boldsymbol{d}=\boldsymbol{Q}\boldsymbol{\eta}, \tag{1}$$

where χ is the surface deformation function, $\boldsymbol{u}\in E$ the translation vector of M, and $\boldsymbol{Q}\in SO(3)$ the proper orthogonal tensor, $\boldsymbol{Q}^T=\boldsymbol{Q}^{-1}$, $\det\boldsymbol{Q}=+1$, representing the work-averaged gross rotation of the shell cross sections from their undeformed shapes described by $(\boldsymbol{x}_{,\alpha},\boldsymbol{\eta})$.

In the shell undergoing phase transition it is assumed that above some level of deformation different material phases A and B may appear in different complementary subregions N_A and N_B separated by the curvilinear phase interface $D\in N$. For a piecewise differentiable mapping χ we can introduce on M a singular image curve $C=\chi^{-1}(D)$ separating the corresponding image regions $M_A=\chi^{-1}(N_A)$ and $M_B=\chi^{-1}(N_B)$. The position vectors of C and D are related by $\boldsymbol{x}_C(s)=\chi^{-1}(\boldsymbol{y}_C(s))$, where s is the arc length parameter along C.

Let us consider a one-parameter family of shell deformations

$$\boldsymbol{y}(\boldsymbol{x},t)=\boldsymbol{x}+\boldsymbol{u}(\boldsymbol{x},t), \quad \boldsymbol{d}_\alpha(\boldsymbol{x},t)=\boldsymbol{Q}(\boldsymbol{x},t)\boldsymbol{x}_{,\alpha}, \quad \boldsymbol{d}(\boldsymbol{x},t)=\boldsymbol{Q}(\boldsymbol{x},t)\boldsymbol{\eta}(\boldsymbol{x}), \tag{2}$$

where t is a time-like scalar parameter such that $t=0$ corresponds to the undeformed placement and t to the deformed one. Then $\upsilon=\dot{\boldsymbol{u}}$ is the virtual translation vector, and $\boldsymbol{\omega}=\mathrm{ax}\,(\dot{\boldsymbol{Q}}\boldsymbol{Q}^T)$ the virtual rotation vector, where $\mathrm{ax}(\ldots)$ is the axial vector associated with the skew tensor $(\ldots)$, $(\dot{\ldots})=d/dt(\ldots)$, while $V=\dot{\boldsymbol{x}}_C\cdot\boldsymbol{v}$ is the virtual translation component in the exterior normal direction of the phase curve C, $\boldsymbol{v}\in T_xM$ is the unit external normal vector to C, and $\boldsymbol{v}\cdot\boldsymbol{\eta}=0$.

In the general resultant theory of shells considered here the following two strain measures are introduced, see Chróścielewski et al. (2004), Eremeyev and Pietraszkiewicz (2004, 2006), and Pietraszkiewicz et al. (2005):

$$\boldsymbol{E}=\boldsymbol{\varepsilon}_\alpha\otimes\boldsymbol{a}^\alpha, \quad \boldsymbol{K}=\boldsymbol{\varkappa}_\alpha\otimes\boldsymbol{a}^\alpha, \quad \boldsymbol{\varepsilon}_\alpha=\boldsymbol{y}_{,\alpha}-\boldsymbol{d}_\alpha, \quad \boldsymbol{\varkappa}_\alpha=\frac{1}{2}\boldsymbol{d}^i\times\boldsymbol{Q}_{,\alpha}\boldsymbol{Q}^T\boldsymbol{d}_i, \tag{3}$$

where $(\boldsymbol{a}^\alpha,\boldsymbol{\eta})$ and $(\boldsymbol{d}^i)$ are bases reciprocal to the base $(\boldsymbol{x}_{,\alpha},\boldsymbol{\eta})$ and the base $(\boldsymbol{d}_\alpha,\boldsymbol{d})$, respectively.

The curvilinear phase interfaces in shells can be either coherent or incoherent in rotations, see Eremeyev and Pietraszkiewicz (2004). For the coherent interface both fields $\boldsymbol{y}$ (or $\boldsymbol{u}$) and $\boldsymbol{Q}$ are supposed to be continuous at C and the kinematic compatibility conditions along C become, see Eremeyev and Pietraszkiewicz (2004), Eqs. (31) and (34),

$$[\![\upsilon]\!]+V[\![\boldsymbol{F}\boldsymbol{v}]\!]=\boldsymbol{0}, \quad [\![\boldsymbol{\omega}]\!]+V[\![\boldsymbol{K}\boldsymbol{v}]\!]=\boldsymbol{0}, \tag{4}$$

where the expression $[\![\ldots]\!]=(\ldots)_B-(\ldots)_A$ means the jump at C.

The phase interface is called incoherent in rotations if only $\boldsymbol{y}$ (or $\boldsymbol{u}$) is continuous at C but the continuity of $\boldsymbol{Q}$ may be violated. In this case the condition $(4)_1$ is still satisfied, but $(4)_2$ may be violated, see Eremeyev and Pietraszkiewicz (2004).

3. Equilibrium equations

The balance equations and corresponding dynamic boundary conditions of the general non-linear theory of shells can be derived exactly by direct through-the-thickness integration of 3D balance laws of linear and angular momentum of