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# Continuity in the plastic strain rate and its influence on texture evolution

Justin C. Mach [a], Armand J. Beaudoin [a,*], Amit Acharya [b]

[a] Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, 1206 W. Green St., Urbana, IL 61801, USA
[b] Department of Civil and Environmental Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, USA

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ABSTRACT

Classical plasticity models evolve state variables in a spatially independent manner through (local) ordinary differential equations, such as in the update of the rotation field in crystal plasticity. A continuity condition is derived for the lattice rotation field from a conservation law for Burgers vector content—a consequence of an averaged field theory of dislocation mechanics. This results in a nonlocal evolution equation for the lattice rotation field. The continuity condition provides a theoretical basis for assumptions of co-rotation models of crystal plasticity. The simulation of lattice rotations and texture evolution provides evidence for the importance of continuity in modeling of classical plasticity. The possibility of predicting continuous fields of lattice rotations with sharp gradients representing non-singular dislocation distributions within rigid viscoplasticity is discussed and computationally demonstrated.



## 1. Introduction

It is well known that Taylor-type material point simulations of texture evolution tend to overestimate intensity and under-represent certain texture components at large deformations. The work of Hirsch and Lücke (1988) provides a comprehensive investigation of this for f.c.c. metals. Maximum intensities tend to be two or more times larger for simulation than experiment. The $\beta$ fiber, which develops in f.c.c. metals undergoing plane strain compression is of particular interest as the prediction of components along this fiber, such as the brass and S components, tend to be weak with respect to the copper and Taylor components. Classical models of texture evolution warrant tremendous merit for their ability to capture general trends in large deformation processes of many metals. However, the subtle inconsistencies mentioned above indicate a fundamental problem in the simulation of orientation evolution and perhaps, as we will show, evolution of plastic flow as a whole.

It will be shown that a certain level of continuity required of the constitutively specified plastic velocity gradient can be interpreted as a continuity condition for the lattice rotation field in a rigid viscoplastic setting. This continuity condition implies a continuous lattice spin field (often referred to as the spin field hereafter) and has been dubbed co-rotation or co-spin. Techniques of this nature have been used in self-consistent material point simulations (Bolmaro et al., 2000; Tomé et al., 2002), also coupled with finite elements (Bolmaro et al., 2006; Signorelli et al., 2006) resulting in textures that are closer to experimental values. This also proved to be useful in predicting recrystallization textures by allowing for grain fragmentation through the co-rotation of neighboring segments of grains (Bolmaro et al., 2005). A commonality among

* Corresponding author. Tel.: +1 217 244 9094; fax: +1 217 244 6534.
*E-mail address:* abeaudoi@illinois.edu (A.J. Beaudoin).

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these works is that the co-rotation methods tend to reduce the intensity of simulated textures and therefore match more closely with experimental results. The co-rotation methods require some or all of the components of the lattice spin of neighboring grains (or parts of grains) to be equal. We will show that this continuity in the spin field is a limiting case of the continuity required of the plastic velocity gradient. We hope to show that models that utilize co-rotation, which are developed from experimental evidence or derived from compatibility or equilibrium equations, are sensible and physically reasonable models.

We will derive a condition of continuity for the spin field, in a finite deformation setting, obtained from a continuity condition associated with a mesocale PDE model of plasticity, referred to as mesoscale field dislocation mechanics (MFDM). MFDM is a result of averaging the evolution equation of excess dislocation (ED) density from the microscopic theory of continuous distribution of dislocations (Acharya and Roy, 2006). The ED density (also referred to as the Nye tensor) is an averaged measure of geometrically necessary dislocations (GNDs), which is in contrast to the statistically stored dislocation (SSD) density which carries no net sign. The ED and SSD densities both contribute to plastic flow, however, the ED density may also contribute to long-range stress effects (Roy and Acharya, 2006; Varadhan, 2007). MFDM provides a precise link between this microscopic theory and classical elastic–plastic modeling of permanent deformation. In doing so, it views the evolution of the Nye tensor as a balance law for Burgers vector content, which implies a partial continuity condition on the plastic rate of deformation at material surfaces of discontinuity; as is well known, conventional plasticity theory poses no restrictions on a surface of discontinuity beyond displacement and traction continuity for quasistatic deformations (these conditions also being included in the new framework). Here, material surfaces have only the requirement that their normal velocity field coincide with the normal component of the velocity of the material particles with which the surface coincides instantaneously. Thus, they could be grain boundaries, but are not required to be so. The new condition has also arisen in the work of Gurtin and Needleman (2005), but not as a consequence of a fundamental balance law, and with a major difference in interpretation in that they assume that the partial continuity is to be automatically expected in solutions of the conventional theory, whereas we do not. It is noteworthy that Mathur and Dawson (1989) used a continuous description of the rotation field which was not motivated by continuity but rather because the rotation was integrated along streamlines in an Eulerian formulation. To ensure that the rotation remained orthogonal they used a two-point integration method with a local interpolation of the Euler angles over finite elements.

It is the intention of the present effort to make a statement pertinent *to the plasticity problem in general* following from a well-founded conservation statement drawn from the continuum theory of distributed dislocations. That is, this work is not to be viewed as an alternative modeling approach adopting gradients in some fashion, but rather as an expression of the boundary value problem. The specific choice of texture evolution in a rigid-viscoplastic material is selected as a specific vehicle for illustrating the implications of the continuity condition through evolution of material state.

An example will help to compare what is implied by conventional plasticity and MFDM. We begin with the fundamental equation of incompatibility, $\boldsymbol{\alpha} = -\operatorname{curl}(\boldsymbol{F}^{e-1})$, where $\boldsymbol{\alpha}$ is the excess dislocation density. Let a grain boundary be oriented perpendicular to the direction $\boldsymbol{e_N}$ with $(\boldsymbol{e_R}, \boldsymbol{e_T}, \boldsymbol{e_N})$ forming a right-handed triad, as shown in Fig. 1. Then, performing standard pill box arguments (cf. Appendix A and Acharya, 2007, Section 4) for pill-surfaces oriented perpendicular to the R and T directions and localizing to the grain boundary, we have

$$\boldsymbol{F}^{e-1}_{+}\boldsymbol{e}_R - \boldsymbol{F}^{e-1}_{-}\boldsymbol{e}_R = \boldsymbol{\alpha}\boldsymbol{e}_T \Rightarrow [\![\boldsymbol{F}^{e-1}]\!]\boldsymbol{e}_R = \boldsymbol{\alpha}\boldsymbol{e}_T$$

$$-\boldsymbol{F}^{e-1}_{+}\boldsymbol{e}_T + \boldsymbol{F}^{e-1}_{-}\boldsymbol{e}_T = \boldsymbol{\alpha}\boldsymbol{e}_R \Rightarrow [\![\boldsymbol{F}^{e-1}]\!]\boldsymbol{e}_T = -\boldsymbol{\alpha}\boldsymbol{e}_R \tag{1}$$

*if the dislocation density field is singularly supported on the grain boundary* and

$$[\![\boldsymbol{F}^{e-1}]\!]\boldsymbol{e}_R = \boldsymbol{0}$$

$$[\![\boldsymbol{F}^{e-1}]\!]\boldsymbol{e}_T = \boldsymbol{0} \tag{2}$$

if the dislocation density field is even in the least bit spread out in the direction perpendicular to the grain boundary. We can write

$$[\![\boldsymbol{F}^{e-1}]\!] = [\![\boldsymbol{F}^{e-1}]\!]\boldsymbol{e}_R \otimes \boldsymbol{e}_R + [\![\boldsymbol{F}^{e-1}]\!]\boldsymbol{e}_T \otimes \boldsymbol{e}_T + [\![\boldsymbol{F}^{e-1}]\!]\boldsymbol{e}_N \otimes \boldsymbol{e}_N \tag{3}$$

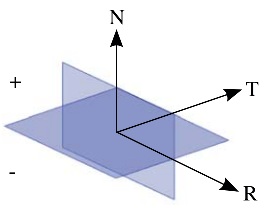


**Fig. 1.** Grain boundary dividing grains ‘ + ’ and ‘ -’ .