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An analytical solution for the galloping stability of a 3 degree-of-freedom system based on quasi-steady theory

Mingzhe He*, John H.G. Macdonald

Department of Civil Engineering, University of Bristol, Bristol BS8 1TR, UK

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ABSTRACT

The aerodynamic forces on a two-dimensional three-degree-of-freedom (3DOF-heave, sway and torsion) body of arbitrary cross-section are considered, for arbitrary wind direction relative to the principal structural axes. The full 3DOF aerodynamic damping matrix is derived, based on quasi-steady theory, using the commonly-used concept of an aerodynamic centre to represent the effect of the torsional velocity on the aerodynamic forces. The aerodynamic coefficients are assumed to be consistent functions of only the relative angle of attack. It is shown that the determinant of the quasi-steady aerodynamic damping matrix is always zero. The galloping stability of the aerodynamically coupled system is then addressed by formulating the eigenvalue problem, for which analytical solutions are derived for the case of perfectly tuned structural natural frequencies. The solutions define a non-dimensional effective aerodynamic damping coefficient, indicating how stable the system is. A trivial solution always exists, with zero effective aerodynamic damping, corresponding to rotation about the aerodynamic centre, and relatively simple exact closed-form solutions are derived for the other one or two solutions, the minimum solution defining the stability of the system. Example results are presented and discussed for square, rectangular (aspect ratio 3) and equilateral triangular sections and a lightly iced cable, and they are compared with results using previous solutions for 2DOF translational and 1DOF pure torsional galloping. For the shapes considered it is found that the stability of the 3DOF system is normally close to that of the 2DOF translational system, with a relatively small influence of the stability of the torsional degree of freedom, although in some instances, especially at the critical angles of attack, it can significantly affect the stability.

1. Introduction

Galloping instability of slender structures has been widely observed in practice, especially for electricity transmission lines, bridge cable stays, etc.. There has been long lasting interest in predicting such dynamic instability by using aerodynamic coefficients in theoretical models, based on quasi-steady theory. The Den Hartog (1932) criterion provides a prediction of galloping instability but is limited to single-degree-of-freedom (1DOF) motion normal to the wind direction, which is not always the case in practice. Aerodynamic coupling between degrees of freedom can be important, especially when the structural natural frequencies are close to each other.

* Corresponding author. Tel.: +44 117 331 5726.
E-mail addresses: mingzhe.he@bristol.ac.uk (M. He), john.macdonald@bristol.ac.uk (J.H.G. Macdonald).

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It is well known that for long span bridges, aircraft wings, etc., flutter instability, i.e. an aeroelastic instability involving torsional motion, is a major concern. Often this is treated as a two-degree-of-freedom (2DOF) problem, involving vertical and torsional motion. For the analysis of flutter of real structures normally numerical approaches are used but Chen and Kareem (2006) proposed a closed-form solution of coupled vertical and torsional flutter for long span bridges, based on flutter derivatives, showing good agreement with experimental results. It should be pointed out that for bridge decks the reduced velocity is quite low resulting in low accuracy of quasi-steady theory. Therefore, flutter derivatives (or equivalent) are commonly used instead. But for smaller diameter bodies, especially cables, the reduced velocity is generally high so quasi-steady theory, as used in conventional Den Hartog galloping, is more applicable, since the timescale of oscillations is much longer than the timescale for the flow to pass the body. In these conditions, as in the present paper which considers a generalisation of quasi-steady theory, 'galloping' is potentially a more appropriate term than 'flutter'.

There is quite a long history of analysing coupled vertical and torsional motions using quasi-steady theory, often called two-degree-of-freedom galloping and often achieving good agreement with experiment results (Blevins and Iwan, 1974, Modi and Slater, 1983, Novak, 1969, Slater, 1969), as reviewed by Blevins (1994). Related to this, Norberg (1993) conducted a series of experiments on rectangular prisms, allowing not only translational motions but also torsion. These were followed up with numerical simulations (Sohankar et al., 1997) which focused on the flow around the cross-section at 0° incidence and the effects of the aspect ratio of the rectangular section, giving some insight into the underlying flow mechanisms driving the instability. Robertson et al. (2003) also performed numerical simulations on rectangular prisms with various side ratios, addressing both the onset of galloping and the steady-state amplitudes of the resulting vibrations. Although the study was conducted at low Reynolds numbers, good agreement was achieved between the numerical results and quasi-steady theory.

Jones (1992) was the first to address coupled translational galloping (across-wind and along-wind), for the special case of perfectly tuned natural frequencies in the two directions. Li et al. (1998) also proposed a series of equations for 2DOF coupled translational galloping, but using body co-ordinates to allow inclination of the wind, which lost the direct connection to the Den Hartog criterion. These and other developments in the analysis of coupled 2DOF translational galloping have recently been critically reviewed by Nikitas and Macdonald (2014). Another useful recent review of various issues in galloping is provided by Piccardo et al. (2014a)

The Den Hartog criterion was generalised to allow for the effects of Reynolds number and any orientation of the cylinder and the plane of 1DOF motion by Macdonald and Larose (2006), for application to dry inclined galloping of bridge stay-cables. The approach was then extended to address the onset of galloping instability of a generalised coupled translational 2DOF model (Macdonald and Larose, 2008a,b), providing a closed-form solution for the minimum structural damping required to prevent galloping in the case of perfectly tuned natural frequencies, and numerical solutions and limit cases for detuned natural frequencies. Meanwhile Carassale et al. (2005) developed an equivalent formulation expressing the aerodynamic damping matrix in terms of vectors and matrix calculus, but excluding Reynolds number effects. These analyses were able to reproduce some of features of dry inclined cable galloping observed in dynamic wind tunnel tests. Also, Luongo and Piccardo (2005) presented an approximate analytical solution for coupled translational galloping, using a perturbation method, which was found to be valid not only in the quasi-resonant condition but also the non-resonant one. Later Piccardo et al. (2011) further investigated the critical conditions for 2DOF translational galloping and explicitly presented the aerodynamic damping matrix of such a system considering both the angle of attack and yaw angle.

There has been a limited amount of research on coupled three-degree-of-freedom (3DOF) galloping. Yu et al. (1993a,b) extended Jones (1992) work by including the torsional degree of freedom, deriving the governing equations using a perturbation approach and solving them for the onset of galloping using the Routh–Hurwitz method. Wang and Lilien (1998) also proposed a 3DOF model, allowing for both single and bundled transmission lines, which was solved using time history simulations. However, both of these 3DOF models focused on the eccentricity and full span effects rather than the aerodynamic coupling between the degrees of freedom and a simplified definition of the conditions for the galloping to occur. Later Luongo et al. (2007), using a linear curved-beam model to analyse galloping, suggested, from order-of-magnitude considerations, that the torsional velocity has negligible influence on the aerodynamic forces. More recently Gjelstrup and Georgakis (2011) developed a 3DOF model based on the 2DOF translational model by Macdonald and Larose (2008a,b) but incorporating torsion. They allowed for variations in Reynolds number and skew angle, as well as angle of attack, which may be relevant in some situations but complicate the formulation considerably. Their solution of the galloping problem was based on the Routh–Hurwitz criterion, which defines whether or not the system is stable but requires significant numerical calculations to quantify the stability. In other developments, Piccardo (1993) addressed coupled aeroelastic phenomena generally and presented the two-dimensional 3DOF quasi-steady aerodynamic damping matrix for certain conditions. Recently, he and his co-workers re-visited the problem and presented the matrix in a more general form (Piccardo et al., 2014a). They have also considered the aeroelastic behaviour of slender tower structures using quasi-steady theory, specifically addressing the dynamic behaviour of the whole structure and nonlinear effects, and presenting some numerical results for the stability Piccardo et al. (2014b). As far as the authors are aware these references cover virtually all the previous analysis of 3DOF galloping.

The aim of the current paper is to address the 3DOF galloping problem, focussing on obtaining a closed-form solution for the effective aerodynamic damping of the system for the first time, albeit for the simplified system of a two-dimensional body with perfectly tuned natural frequencies in the three structural modes. Such a system may be particularly relevant to bundled conductors, which have close natural frequencies for both torsional and translational motions (Chabart and Lilien,