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Drag reduction by elastic reconfiguration of non-uniform beams in non-uniform flows

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ABSTRACT

Flexible systems bending in steady flows are known to experience a lesser drag compared to their rigid counterpart. Through a careful dimensional analysis, an analytical expression of the Vogel exponent quantifying this reduction of drag is derived for cantilever beams, within a framework based on spatial self-similar modelling of the flow and structural properties at the clamped edge of the structure. Numerical computations are performed on various situations, including systems involving more complex distributions of the flow or structural parameters. The scaling of drag versus flow velocity for large loadings is shown to be well predicted by fitting the system properties by simple power laws at the scale of the length on which significant bending occurs. Ultimately, the weak sensitivity of the Vogel exponent to the parameters of the system provides an explanation to the rather reduced scattering of the Vogel exponents around -1 observed on most natural systems in aquatic or aerial vegetation.

1. Introduction

It has been well known, since the seminal work of Vogel (1984), that flexible structures subjected to fast flows experience a drag F that grows slower with the velocity than if they were rigid. When the velocity U of the flow exceeds a given threshold, the classical quadratic velocity–drag law that holds for rigid bodies at large Reynolds number changes to a smaller power law $F \propto U^{2+\nu}$ characterized by the so-called Vogel exponent ν, which is negative. This phenomenon is for instance broadly observed in nature. Indeed, plants growing in fast flow environments are very often made of flexible tissues that bend to comply with the flow, hence lowering the risk of failure by fracture or uprooting.

To get a better understanding of the underlying mechanisms, Alben et al. (2002, 2004) first studied the model problem of an elastic one dimensional fibre in an inviscid two dimensional flow, both experimentally and numerically. Their study revealed the importance of a single control parameter, which they call the elastohydrodynamical number, related to the more commonly used Cauchy number C_Y (Tickner and Sacks, 1969; Chakrabarti, 2002; de Langre, 2008), that scales the competing effects of fluid loading to the elastic restoring force. The model of Alben et al. (2002, 2004) exhibits the expected transition from the classical rigid-body U^2 drag scaling law to a new $U^{4/3}$ drag law concomitant with the convergence towards a self-similar shape at large Cauchy numbers. Gosselin et al. (2010) obtained similar results for a finite width plate with a simplified model of fluid loading. They also managed to predict the same asymptotic drag scaling law from a very simple dimensional analysis. In order to explain the drag reduction due to the rolling up of daffodil leaves originally

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observed by Vogel (1984), Schouveiler and Boudaoud (2006) obtained theoretical and experimental estimates of the Vogel exponent for circular plastic sheets cut along a radius. They found a drag scaling as $U^{2/3}$, while a theoretical and numerical study by Alben (2010) on the same system concludes that the drag increases as U^1. Recent studies have proposed models that account for additional effects such as gravity (Luhar and Nepf, 2011), viscosity (Zhu and Peskin, 2007; Zhu, 2008), shear background flow (Henriquez and Barrero-Gil, 2014) or unsteady wake effects due to vortex shedding at the edges (Yang et al., 2014).

Many experimental measurements made either in the field or in the laboratory have also been able to provide estimates of the Vogel exponents for systems as diverse as full trees, grasses, flowers, leaves, near-shore marine macrophytes or freshwater algae. Some quite comprehensive reviews such as Harder et al. (2004) or de Langre et al. (2012) list Vogel exponents varying in a range around -0.7, between 0 and -1.3 at most for such systems. What is especially striking is not the scattering of Vogel exponents found for different systems, but much more the robustness of the drag reduction phenomenon with respect to the great variability of structural and flow configurations, and the rather narrow range in which the Vogel exponents usually lie. From the assumption that the scaling of drag reduction results from the loss of one typical length scale, de Langre et al. (2012) showed that the Vogel exponent of any structure made of beams and plates (such as most plants) should exhibit approximately the same behaviour. By a simple dimensional analysis, they recovered the classical $-2/3$ Vogel exponent found by Alben et al. (2002, 2004) and Gosselin et al. (2010). They further claimed that non-linearity in the material constitutive law should have little impact on the scaling of drag. Any possible effect of flow or structural non-uniformities was however not addressed in this study, and nor was it, to the authors' knowledge, in any other one, with the only exception of Henriquez and Barrero-Gil (2014) in the specific case of shear flow. A range of models is clearly missing to fill the gap between the idealized cases above and the more complex natural configurations.

The goal of the present work is to provide a general framework for the derivation of the Vogel exponent of a flexible beam in the limit of large velocity flows. It includes most possible non-uniformities in the flow or structural parameters, but it excludes the additional effects of viscosity, unsteadiness in the wake or in the background flow, or other external forces such as gravity. In some aspects, it is a generalization of the works of Gosselin et al. (2010), de Langre et al. (2012) and Luhar and Nepf (2011).

In Section 2, the general framework of this study is described. In Section 3, a theoretical analysis of drag reduction of a system described by self-similar fluid and structural parameters is presented. In Section 4, the results of numerical simulations performed on several practical cases are given. Finally, Section 5 discusses the implications of the present results regarding the understanding and the predictability of the typical values of the Vogel exponents of actual systems. A nomenclature of the main variables used throughout this paper is given in Table 1.

2. Model

The model used in this paper is represented in Fig. 1. The elastic body is a cantilever beam of length L bending in the xz-plane. The width W, thickness D and material stiffness may all vary with the curvilinear coordinate s. The height $z(s)$ and

Table 1
Nomenclature.

L, $W(s)$, $D(s)$	Length, width and thickness of the beam
$EI(s)$	Bending stiffness in the case of linear elasticity
$C_D(s)$	Cross-section drag coefficient
$\rho(z)$	Fluid density distribution
$U(z)$, U_0	Flow profile and reference velocity
$\theta(s)$, $\kappa(s)$	Inclination angle of the beam from the vertical axis and curvature
$m(s)$, m_0	Internal bending moment and reference value
$f_n(s)$	Internal shear force
$q(s)$, q_0	Local normal fluid load and reference value
$c(\theta)$	Angular dependence of the normal fluid load
$g(\kappa)$, α	Function and exponent associated with the material constitutive law
$b(s)$, b_0, β	Distribution, reference and exponent associated with the stiffness factor
$w(s)$, w_0, γ	Distribution, reference and exponent associated with the cross-section shape factor
$p(z)$, p_0, μ	Distribution, reference and exponent associated with the pressure
ϕ, ψ	Geometrical and material parameter
F, F_{rigid}	Drag force on the flexible/rigid beam
$\mathcal{R}$	Reconfiguration number
C_Y	Cauchy number
ν, ν_∞	Local and asymptotic Vogel exponents
ℓ	Characteristic non-dimensional bending length
L_B	Characteristic non-dimensional boundary layer thickness
δ	Characteristic non-dimensional tapering length