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Shock response of a system of two submerged co-axial cylindrical shells coupled by the inter-shell fluid

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ABSTRACT

The shock response of a submerged system consisting of two co-axial cylindrical shells coupled with the fluid filling the inter-shell space is considered. The shock–structure interaction is modeled using a semi-analytical methodology based on the use of the classical apparatus of mathematical physics. Both the fluid and structural dynamics of the interaction is addressed, with special attention paid to the interplay between the two. It is demonstrated that the wave effects due to multiple reflections of the pressure waves travelling in the inter-shell fluid to a large degree determine the structural dynamics of the system, but have a more pronounced effect on the outer shell than on the inner one. It is also established that the effect of changing the thickness of the outer shell on the stress–strain state of the inner shell is incomparably more pronounced than vice versa. The investigation culminates with the results of a parametric study of the overall peak stress in the system, an example of utilizing the approach developed based on the introduced model and aiming at facilitating structural optimization of industrial systems at the pre-design stage in the context of shock resistance.

1. Introduction

Systems that can be represented by two co-axial circular cylindrical shells with fluid in between the shells are relatively common in industry, with submersible vehicles of certain designs and heat exchange systems being the most obvious examples. More often than not, the issue of shock resistance of such systems is of importance, and in some cases it is one of the most important considerations during the design process. At the same time, the response of a two-shell configuration to a shock or other non-stationary loading appears to have been addressed significantly less extensively than the classical single-shell configuration, with a multitude of studies spanning six decades existing for the latter (e.g., Mindlin and Bleich, 1953; Haywood, 1958; Geers, 1969; Huang and Wang, 1970; Huang, 1975; Takano et al., 1997; Gregson et al., 2006; Iakovlev, 2008a; Iakovlev, 2008a,b).

One of the best-known studies specifically devoted to the double-shell configuration is an analytical work dating back to the late 1970s (Huang, 1979a) in which the response of a submerged system of two co-axial cylindrical shells with fluid filling the inter-shell space to an external shock loading is considered, and which has been a definitive benchmark for decades (e.g., Mair, 1999). The study provided answers to a number of fundamental questions about the interaction with a

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Nomenclature

a	radius of the inner shell, $\hat{a} = ar_0^{-1}$
c_i	sound speed in the inter-shell fluid, $\hat{c}_i = c_i c_e^{-1}$
c_e	sound speed in the external fluid, $\hat{c}_e = 1$
c_1	sound speed in the material of the inner shell, $\hat{c}_1 = c_1 c_e^{-1}$
c_2	sound speed in the material of the outer shell, $\hat{c}_2 = c_2 c_e^{-1}$
E_1	Young modulus of the material of the inner shell, $\hat{E}_1 = E_1 \rho_e^{-1} c_e^{-2}$
E_2	Young modulus of the material of the outer shell, $\hat{E}_2 = E_2 \rho_e^{-1} c_e^{-2}$
h_1	thickness of the inner shell, $\hat{h}_1 = h_1 r_0^{-1}$
h_1	thickness of the outer shell, $\hat{h}_2 = h_2 r_0^{-1}$
I_n	modified Bessel function of the first kind of order n
J_n	Bessel function of the first kind of order n
K_n	modified Bessel function of the second kind of order n
p_α	peak incident pressure, $\hat{p}_\alpha = p_\alpha \rho_e^{-1} c_e^{-2}$
p_{1s}	total pressure on the surface of the inner shell, $\hat{p}_1^s = p_1^s \rho_e^{-1} c_e^{-2}$
p_2^s	total pressure on the surface of the outer shell, $\hat{p}_2^s = p_2^s \rho_e^{-1} c_e^{-2}$
p_0	incident pressure, $\hat{p}_0 = p_0 \rho_e^{-1} c_e^{-2}$
p_d	diffraction pressure, $\hat{p}_d = p_d \rho_e^{-1} c_e^{-2}$
p_r^e	external radiation pressure, $\hat{p}_r^e = p_r^e \rho_e^{-1} c_e^{-2}$
p_r^i	inter-shell radiation pressure, $\hat{p}_r^i = p_r^i \rho_e^{-1} c_e^{-2}$
r	radial coordinate of the polar coordinate system, $r = \varrho r_0^{-1}$
r_0	radius of the outer shell, $\hat{r}_0 = 1$
R_0	radial distance to the source of the shock wave, $\hat{R}_0 = R_0 r_0^{-1}$
S_R	shock wave stand-off, $\hat{S}_R = S_R r_0^{-1}$
t	time, $t = \tau c_e r_0^{-1}$
v_1^*	transverse displacement of the middle surface of the inner shell, $v_1 = v_1^* r_0^{-1}$
v_2^*	transverse displacement of the middle surface of the outer shell, $v_2 = v_2^* r_0^{-1}$
w_1^*	normal displacement of the middle surface of the inner shell, $w_1 = w_1^* r_0^{-1}$
w_2^*	normal displacement of the middle surface of the outer shell, $w_2 = w_2^* r_0^{-1}$
Y_n	Bessel function of the second kind of order n
θ	angular coordinate of the polar coordinate system
λ	exponential decay rate, $\hat{\lambda} = \lambda c_e r_0^{-1}$
ν_1	Poisson's ratio of the material of the inner shell
ν_2	Poisson's ratio of the material of the outer shell
ξ_n^e	external response functions
ξ_n^1	first inter-shell response functions
ξ_n^2	second inter-shell response functions
ρ_i	density of the inter-shell fluid, $\hat{\rho}_i = \rho_i \rho_e^{-1}$
ρ_e	density of the external fluid, $\hat{\rho}_e = 1$
ρ_1	density of the material of the inner shell, $\hat{\rho}_1 = \rho_1 \rho_e^{-1}$
ρ_2	density of the material of the outer shell, $\hat{\rho}_2 = \rho_2 \rho_e^{-1}$
ϱ	radial coordinate of the polar coordinate system, $r = \varrho r_0^{-1}$
$\sigma_{\theta\theta}^1$	transverse stress in the inner shell, $\hat{\sigma}_{\theta\theta}^1 = \sigma_{\theta\theta}^1 \rho_e^{-1} c_e^{-2}$
$\sigma_{\theta\theta}^2$	transverse stress in the outer shell, $\hat{\sigma}_{\theta\theta}^2 = \sigma_{\theta\theta}^2 \rho_e^{-1} c_e^{-2}$
τ	time, $t = \tau c_f r_0^{-1}$
ϕ	total fluid velocity potential, $\hat{\phi} = \phi c_e^{-1} r_0^{-1}$
ϕ_0	fluid velocity potential in the incident wave, $\hat{\phi}_0 = \phi_0 c_e^{-1} r_0^{-1}$
ϕ_d	fluid velocity potential in the diffracted wave, $\hat{\phi}_d = \phi_d c_e^{-1} r_0^{-1}$
ϕ_r^e	fluid velocity potential in the external radiated wave, $\hat{\phi}_r^e = \phi_r^e c_e^{-1} r_0^{-1}$
ϕ_r^i	fluid velocity potential in the inter-shell radiated wave, $\hat{\phi}_r^i = \phi_r^i c_e^{-1} r_0^{-1}$

$(*)_n \sin n\theta$ and $(*)_n \cos n\theta$ denote the harmonics of $(*)$. Unless stated otherwise, capitalized symbols denote the Laplace transforms of the corresponding functions. Other symbols are defined in the text.

two-shell system, but only focused on the structural dynamics of the system; at the same time, it is the fluid dynamic effects that determine the complexity of the stress–strain state that is seen in the system. A closely related study of the response of two concentric spherical shells was also offered by the same author (Huang, 1979b).

The shock response of a double-hull design was also addressed in an experimental investigation by Stultz et al. (1994) where a system consisting of a two-shell arrangement with additional weight rings was considered, a model intended to provide a more realistic representation of certain types of industrial systems; the results were compared with the outcomes of respective numerical simulations. The time-histories of the structural strains and velocities were recorded along with those for the pressure in the inter-shell domain, and a number of insights into the fundamental physics of the shock response of a two-shell configuration were offered, notably the remarks on the influence of the outer shell on the structural dynamics of the inner one. The hydrodynamic fields induced during the interaction were not addressed.

Another highly relevant study is the investigation by Wardlaw and Luton (2000) where an explosion inside a double-wall cylinder was considered, essentially addressing the 'internal' version of the present problem for the scenario where the inter-shell clearance is small. Both fluid and structural dynamics of the interaction were considered, with a very informative discussion presented of the mutual influence of the fluid dynamic and structural effects; particular attention was paid to the