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Parametric vibrations of flexible hoses excited by a pulsating fluid flow, Part I: Modelling, solution method and simulation

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ABSTRACT

The article presents an analysis of a model describing lateral vibrations of a pipe induced by fluid flow velocity pulsation. The motion has been described with a set of two non-linear partial differential equations with periodically variable coefficients. In the analysis Galerkin method has been applied using orthogonal polynomials as shape function. To determine instability regions Floquet theory has been employed. The effect of selected parameters on parametric resonance ranges and regions of increased vibration level has been investigated. The character and form of vibrations have been investigated indicating the possibility of excitation of sub-harmonic and quasi-periodic vibrations in the combination resonance ranges.

1. Introduction

The machine power transmission hydraulic system usually consists of an engine driven pump together with a set of valves, with protection and control function, and operational elements such as hydraulic cylinders, hydraulic motors, etc. The pipes in such systems transmit power between the particular sub-assemblies. The pipes used are both rigid – steel pipes and flexible – rubber hoses. The rigid pipes are used to connect elements whose position relative to each other does not change (e.g. fixed on a common supporting structure). However, frequently some element of hydraulic system (e.g. hydraulic cylinder) changes its position during the operation and then it is necessary to apply a flexible connection. In such cases, due to the high pressure of the fluids, flexible hoses made from steel braided synthetic rubber are used. The composite structure of the hose ensures the resistance to inner pressure while high flexibility is maintained.

One of the main causes of vibrations in hydraulic systems, including vibrations of the pipes, is time variable fluid flow rate. On the one hand it is a periodic flow pulsation resulting from pump non-uniform delivery or intentional flow rate control. On the other hand, sudden variations of flow rate may occur resulting from fast opening or closing the valves, which can result in water hammer effect. Vibrations can cause noise emission, pipes life loss, weakening of joints or other threats for a hydraulic system.

For certain values of flow parameters (velocity, pressure) larger than the so-called critical values, there may occur conditions for pipe stability loss due to buckling (Holmes, 1977, 1978). Critical values depend mainly on pipe geometry,

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Nomenclature

a	dimensionless flow pulsation amplitude
A, A_p	internal sectional area and cross-sectional area of a pipe
$\mathbf{A}$	state-transition matrix
$\mathbf{B}$, $\mathbf{C}$	stiffness and damping matrix
B^y_{ni}, C^y_{ni}	linear coefficients of Eq. (33)
B^z_{ni}, C^z_{ni}	linear coefficients of Eq. (34)
B^{zz}_{nij}, C^{zz}_{nij}	non-linear coefficients of Eq. (33)
B^{zy}_{nij}, B^{yz}_{nij}, B^{zzz}_{nijk}	dissipative non-linear coefficients of Eq. (34)
C^{zy}_{nij}, C^{zzz}_{nijk}	other non-linear coefficients of Eq. (34)
E, E_0	Young modulus, Young modulus for $\theta=\theta_0$ and $p_0=0$
$\mathbf{F}$	fundamental matrix
g	gravity acceleration due to gravity
I_p	cross section moment of inertia
L	length of a pipe
m_f, m_p, m	elementary mass of fluid, mass of a pipe, mass of the hose with fluid
M	bending moment
$\mathbf{M}$	monodromy matrix
p, p_0	pressure inside the hose, pressure for $x=L$
Q	transverse force
$\mathrm{r}_{n\text{-}m}$, r_n	principal resonance, main secondary resonance
t, τ	time, dimensionless time
T, T_0	axial force, axial force for $x=L$
T_S	initial tension force
T_p	dimensionless excitation period
$u(x,t)$	dimensional axial displacement
$U(\tau)$, U_0	dimensionless flow velocity and average flow velocity
$U_f(t)$, U_{f0}	flow velocity, average flow velocity
$\mathbf{v}$	state vector
$v_{1/4}$, $v_{1/2}$	dimensionless vibration velocity in points $x=L/4$ and $x=L/2$
$w(x,t)$	dimensional transverse displacement
$w_0(t)$	kinematic excitation
x, ξ	dimensional and dimensionless coordinate along pipe
$y(\xi,\tau)$, $y_n(\tau)$	dimensionless axial displacement
$z(\xi,\tau)$, $z_n(\tau)$	dimensionless transverse displacement
$\mathbf{z}$	transverse displacement vector
α	internal damping coefficient
β	mass ratio
Γ	dimensionless parameter
ε	axial strain
ς	dimensionless internal damping coefficient
θ	temperature
θ_v	vibration index
κ	dimensionless stiffness ratio
μ, $\mu_{\max}$	Floquet multiplier, maximal Floquet multiplier
ν	Poisson ratio
ρ	dimensionless inertia ratio
φ	angle between x-axis and tangent to the centerline of the pipe
$\varphi_n(\xi)$, $\overline{\phi}_n(\xi)$	orthonormal and base polynomial functions for transverse displacement
$\psi_n(\xi)$, $\overline{\psi}_n(\xi)$	orthonormal and base polynomial functions for axial displacement
Ω	pulsation frequency
ω_0	reference frequency
ω_n	dimensionless natural frequency
ω_p	dimensionless pulsation frequency
ω_r	dimensionless first natural frequency

physical properties of the material and manner of pipe support. At lower values of flow, parametric stability loss can also take place leading to parametric resonance excitation (Gregory and Païdoussis, 1966a,1966b). For parametric resonance to occur the velocity of the fluid flowing through the pipe must have a component periodically variable in time. At certain values of pulsation frequency and pulsation amplitude high enough as well as high flow velocity parametric resonance of considerable amplitudes are generated.

The dynamics of systems of continuous mass distribution, such as beams, conveyor belts or pipes conveying fluids, is often described with non-linear partial differential equations with respective boundary and initial conditions. In a general case the solution of such equations by exact methods is not possible. It is necessary to develop approximate methods of analysis. The results obtained this way should be verified experimentally.

The first researchers to carry out a correct analysis of a linear model of a pipe with pulsating fluid flow were: Ginsberg (1973) for a pipe simply supported at both ends, Païdoussis and Issid (1974) for a cantilevered pipe and Païdoussis and Sundararajan (1975) for a clamped-clamped pipe. The analytical results were confirmed through an experiment (Païdoussis and Issid, 1976).

Semler et al. (1994) made a comparison of the assumptions adopted in the major studies on non-linear models of pipes supported at one or both ends. They also provided discussion on the correctness of the equations obtained by various authors. Païdoussis gave a comprehensive review of the modelling methods and analysis in his monograph (1998).

The problem of non-linear dynamics of conduits with pulsating flow was studied by many scientists. The first to perform a non-linear analysis of parametric resonance were Yoshizawa et al. (1986), Namachchivaya (1989) and Namachchivaya and Tien (1989). They employed analytic methods to study the effect of the system's parameters on principal simple and combination resonance ranges. Jayaraman and Narayanan (1996) determined the ranges of chaotic vibrations. Gorman et al. (2000) studied a non-linear model of a pipeline using the finite difference method and the method of characteristics, and determined flow critical parameters (velocity and hydrodynamic pressure). Öz and Boyaci (2000) determined analytically