Journal of Fluids and Structures 55 (2015) 204–217

Contents lists available at ScienceDirect

Journal of Fluids and Structures

journal homepage: www.elsevier.com/locate/jfs

Dynamics and stability of a flexible pinned-free cylinder in axial flow

M. Kheiri*, M.P. Païdoussis

Department of Mechanical Engineering, McGill University, 817 Sherbrooke Street West, Montreal, QC, Canada H3A 0C3

ARTICLE INFO

Article history:
Received 20 August 2014
Accepted 26 February 2015
Available online 21 April 2015

Keywords:
Pinned-free cylinder
Stability
Dynamics
Axial flow

ABSTRACT

In this paper, the extended Hamilton's principle is used to obtain the linear equation of motion and boundary conditions for a cylinder flexibly supported by a translational and a rotational spring at the upstream end and free at the other, and subjected to axial flow. The equation of motion is solved numerically via Galerkin's method for a system in which the stiffness of the translational spring is infinitely large, while that of the rotational spring is zero, i.e. a pinned-free cylinder. For such a system, the condition for occurrence of non-oscillatory rigid-body instability is obtained analytically. Also, the Adomian Decomposition Method is used to obtain the critical flow velocity for divergence of pinned-free cylinders analytically. Finally, previously obtained experimental results for pinned-free cylinders are compared with those obtained numerically using the present theory.

1. Introduction

The study of dynamics and stability of flexible slender structures in axial flow has generated a great interest among applied mechanics researchers to-date. Many studies have been made, some of which are concerned about specific applications, while most are curiosity-driven. The first *serious* study of the system dates back to the late 1950s and was undertaken by Hawthorne (1961), who proposed the first basic model for the dynamics of a flexible sausage-like tubular body, the so-called *Dracone*. Later, a series of theoretical and experimental studies were carried out by Païdoussis (1966a,b, 1968, 1970, 1973) on the dynamics of flexible cylindrical structures with various boundary conditions, such as clamped–free, simply-supported and towed-free. For clamped–free and supported-ends cylinders, it was shown that, for sufficiently high flow velocities, the cylinder may be subject to divergence (buckling) in its first mode and to oscillatory instabilities (i.e. flutter) in higher flexural modes. For a towed flexible cylinder, on the other hand, it was found that, the cylinder is generally prone to both rigid-body and flexural instabilities, the former occurring at relatively low towing speeds, while the latter develop at higher speeds; refer to Païdoussis (2004) for a more extensive review.

The emergence of new applications for towed flexible cylinders, e.g. towed 'acoustic streamers', has motivated more recent work in this area; some examples are the studies by Triantafyllou and Chryssostomidis (1984, 1985) on the dynamics of a clamped–free beam and a pinned-free string and the studies by Dowling (1988a,b) on the dynamics of neutrally and negatively buoyant elements of a towed system, i.e. the cylinder itself and the towing cable, respectively. These studies in which the flexural rigidity of the body was either neglected altogether (Triantafyllou and Chryssostomidis, 1985) or only

* Corresponding author. Tel.: +1 604 716 4623; fax: +1 514 398 7365.
E-mail address: mojtaba.kheiri@mail.mcgill.ca (M. Kheiri).

http://dx.doi.org/10.1016/j.jfluidstructs.2015.02.013
0889-9746/

Nomenclature

A	cylinder cross-sectional area
C_D	zero-flow normal coefficient
C_T	longitudinal friction drag coefficient
C_N	normal friction drag coefficient
C_{TD}	form-drag coefficient for the end-piece
D	cylinder diameter
EI	cylinder flexural rigidity
F_A	inviscid hydrodynamic force per unit length
F_L	viscous force per unit length in the longitudinal direction
F_N	viscous force per unit length in the normal direction
f	correction factor for inviscid hydrodynamic force over the end-piece
L	cylinder length
ℓ	end-piece length
M	fluid added mass per unit length
m	cylinder mass per unit length
s	curvilinear coordinate along the cylinder centreline
t	time
U	mean flow velocity
x	axial coordinate
y	lateral coordinate
θ	angle between the centreline of the deformed body and the x-axis
ρ	fluid density

partly taken into account (Dowling, 1988a) concluded that a typical acoustic streamer is *stable* at all towing speeds, a conclusion which today we know is incorrect (see, for example, de Langre et al., 2007).

Recently, more advanced, systematic theoretical/experimental studies have been undertaken on the dynamics of flexible cylindrical structures in axial flow. More specifically, in these studies, the previous linear models have been used for revisiting the problem, e.g. by de Langre et al. (2007) and Kheiri and Païdoussis (2011), or for analyzing the dynamics of systems in new applications, e.g. by Kheiri et al. (2013b,c); moreover, some nonlinear models have been developed, e.g. by Lopes et al. (2002), Modarres-Sadeghi et al. (2005), Kheiri et al. (2013a), and new experiments have been conducted, e.g. by Sudarsan et al. (1997), Païdoussis et al. (2002), Modarres-Sadeghi (2006) and Kheiri et al. (2014a).

In this paper, for the first time, a theoretical model is developed for the dynamics of a flexible, slender cylinder flexibly supported (i.e. spring-supported) at its upstream end and free downstream. It is, in fact, of fundamental importance to study the transition in dynamical behaviour between a well-supported cylinder (e.g. clamped–free) and one supported in a flimsy way, which represents an intermediate but interesting state, approaching a free–free system. The theoretical model developed in this paper will pave the way for conducting such studies. Here, the dynamics and stability of a pinned-free cylinder in axial flow is investigated as a particular case in which there is also a more practical interest: a pinned-free cylinder may be regarded as a limit case of a towed system, as the towrope becomes very short; this has been done by others before (e.g. Triantafyllou and Chryssostomidis, 1985) but in a simplified manner and in some cases based on flawed assumptions. It is noted that an exhaustive study is not the intent of this paper; instead, in addition to presenting a versatile theoretical model, this paper gives the reader an insight on the dynamics of a pinned-free cylinder in axial flow.

The rest of the paper is organized as follows: in Section 2, the equation of motion and boundary conditions are obtained for a cylinder flexibly restrained by a translational and a rotational spring at the upstream end and free at the other, and subjected to axial flow. In Section 3, using the derived equations, some numerical and analytical analyses are performed for pinned-free cylinders, via which the dynamics of such systems is illustrated. In Section 4, some numerical results obtained with the present theoretical model are compared with experimental results for pinned-free cylinders, previously obtained by Païdoussis (refer to Païdoussis, 2004, Section 8.9.4).

2. Theory

2.1. Definitions and preliminaries

The system under consideration comprises a flexible cylinder of length L and cross-sectional area A, which is supported at one end by a translational and a rotational spring and is free at the other. At the free end, the cylinder is terminated by a short tapering piece (of length $\ell \ll L$) which is assumed to be rigid. The body is immersed in an incompressible fluid of density ρ, flowing with uniform velocity U parallel to the x-axis, which coincides with the position of rest of the body (see Fig. 1). Except for the tapering end-piece, the body is of constant mass per unit length m and flexural rigidity EI; it is further assumed that the centreline of the cylinder is inextensible[1] and that the body is neutrally buoyant (i.e. $\beta = 0.5$, β being the ratio of virtual fluid mass[2] to the sum of virtual fluid mass and cylinder mass), thus the forces due to gravity and buoyancy do not come into play.

[1] Inextensibility of the centreline means that the distance between two arbitrary points on the centreline remains constant before and after deformation. In other words, the axial strain due to oscillations is regarded to be negligible.

[2] A *virtual* or *added* mass is commonly defined, by expressing the fluid loading in the form of a d'Alambert (mass) × (acceleration) term. For a long cylinder oscillating in unconfined fluid, for example, it is found that the virtual fluid mass is equal to the displaced mass of fluid (refer to Païdoussis, 2014, Chapter 2).