Journal of Fluids and Structures 55 (2015) 324–346

Contents lists available at ScienceDirect

Journal of Fluids and Structures

journal homepage: www.elsevier.com/locate/jfs

Piecewise constrained optimization harmonic balance method for predicting the limit cycle oscillations of an airfoil with various nonlinear structures

Haitao Liao *

Chinese Aeronautical Establishment, Beijing 100012, China

A R T I C L E   I N F O

Article history:
Received 17 November 2014
Accepted 9 March 2015
Available online 21 April 2015

Keywords:
Piecewise nonlinearity
Limit cycle
Harmonic balance method
Sensitivity

A B S T R A C T

A method is proposed to calculate the periodic solutions of piecewise nonlinear systems. The method is based on analytical derivation of nonlinear multi-harmonic equations of motion. Since periodic variations of nonlinear forces are characterized by different states, the vibration cycle is broken into sequential transition intervals according to the instant sets of state transitions. Analytical formulations of the harmonic coefficients of the nonlinear forces and its derivatives with respect to the harmonic coefficients of displacements are developed. Sensitivities of the harmonic coefficients of periodic solutions are determined for constructing explicit expressions for vibration amplitude levels as a function of structural parameters. Numerical investigations of the limit cycle oscillations and its sensitivities of an airfoil with different piecewise nonlinearities have been performed. The results show that the developed method is capable of determining the periodic solutions and its sensitivities with respect to the structural parameters. In order to guarantee time continuity of the nonlinear force, for the hysteresis model it is not right to track the periodic solutions by using the preload or freeplay as the continuation parameters.

1. Introduction

The influence of structural nonlinearities such as cubic, freeplay and hysteresis on the aeroelastic response has received significant attention in aerospace community (Lee et al., 2005). The two-dimensional wing section (Liu and Dowell, 2004; Liu et al., 2007), restrained by two springs in plunge and pitch motions, is a common structural model that is widely used for the aeroelastic modeling. Extensive investigations have been carried out on the aeroelastic response of cubic nonlinearity. Although the analysis of the dynamic behavior in aeroelastic systems with freeplay and hysteresis is hampered by the non-smooth nonlinearity feature, research on these kinds of non-smooth dynamic systems has also been performed. For example, Zhang et al. (2013) studied the limit cycle oscillations (LCOs) of an airfoil with torsional freeplay and the effects of the freeplay value and contact stiffness ratio on the frequency, amplitude and phase of LCO are revealed. The averaging method and the Floquet theory are employed by Guo and Chen (2012) to investigate the LCOs of a 2-D of airfoil containing a combination of freeplay and cubic nonlinearities in supersonic flow. It has been demonstrated that the presence of freeplay

* Tel.: +86 10 15810533731.
E-mail addresses: liaoht@cae.ac.cn, ht0819@163.com

http://dx.doi.org/10.1016/j.jfluidstructs.2015.03.008
0889-9746/

nonlinearity modifies the aeroelastic behaviors, resulting in interesting phenomena such as amplitude jump. In addition, it should be noted that there have been some studies for the analysis of aeroelastic dynamics under uncertainties, such as the work of Liao (2013).

Several numerical methods have been developed to treat the piecewise nonlinearity airfoil system. In several works of Liu et al. (2002a, 2002b), a mathematical technique based on the point transformation method is presented to investigate the aeroelastic model with a freeplay nonlinearity as well as a hysteresis nonlinearity. Two formulations are developed to detect the periodic motions with harmonics, period doubling, chaotic motions and the coexistence of stable limit cycles.

Moreover, the perturbation incremental method is introduced by Chung et al. (2007, 2009) to deal with structural nonlinearities like freeplay and hysteresis. With the help of the time discretization technique of the cyclic method, the incremental harmonic balance technique in conjunction with the Fourier expansion approach of hysteresis nonlinearity is used by Liu et al. (2012) to simulate the dynamics of an airfoil with a pitching hysteresis nonlinearity. In Chen and Liu (2014) and Cui et al. (2015), bifurcation and LCOs of the aeroelastic systems with freeplay nonlinearity are analyzed and a predictor–corrector algorithm based on the precise integration method is proposed to determine the switching points of the piecewise nonlinearity system. Chaotic response of a two-Dof airfoil in subsonic flow with structural nonlinearities was predicted by Li et al. (2012). The rational polynomials have been introduced to properly approximate both the freeplay and hysteresis nonlinearities. To overcome these computational limitations for approximation of the nonlinear forces in Li et al. (2012) and Liu et al. (2012), a new approach using piecewise integration is proposed in the present contribution.

A number of investigators have contributed to the development of the harmonic balance method(HBM) or have used the technique to solve LCOs of wings and cascades. The HBM expands periodic response of a nonlinear system as truncated Fourier series, whose coefficients are determined by solving a set of nonlinear algebraic equations. Recently, with the aid of the symbolic calculation software Mathematica, the complex Fourier coefficients associated with geometric nonlinearity term is derived by Dai et al. (2014) and a fast HBM based on the optimal iterative algorithm and explicit Jacobian matrix has been developed to produce periodic solutions of an airfoil with cubic nonlinearity.

Continuation of periodic orbits has now been recognized as the most efficient method for numerically computing the family of periodic orbits and their implementation is well documented within numerous standard methods for step continuation and adaptation (arclength, pseudo-arclength) (Sarrouy and Sinou, 2011). However, if some branches of solutions are disconnected (in the sense that they do not arise from bifurcation points of previous branches), then the continuation algorithm would fail to detect them and new tools would be needed to solve the algebraic system of equations induced by the HBM.

A method named constrained optimization harmonic balance method (COHBM) has been developed by Liao (2013) and Liao and Sun (2013) for the numerical prediction of the periodic solutions of nonlinear mechanical systems. The method is essentially a nonlinear constrained optimization problem and consists of a two-step procedure: In the first step, the harmonic balance method and the stability Hill method are used to construct the nonlinear equality and inequality constraints. In the second step, the MultiStart optimization algorithm is used to find the optimization solution. The proposed methodology is applied to several nonlinear mechanical systems.

Sensitivity analysis (Narimani et al., 2004) for the determination of the gradients of objective and constraints is the dominant process in the accuracy and computational time of many optimization problems. In addition, the sensitivity analysis provides the trends of variation of the objective and constraint functions versus system parameters efficiently. Therefore, the sensitivity analysis might be used as a useful design tool to evaluate the system response to changing parameters. Numerical methods such as finite difference technique have been used to determine the gradients of the objective and constraints function. However, finite difference evaluation of gradients may result in inaccurate optimal solution and is also computationally expensive. Therefore, an efficient analytical evaluation of sensitivity gradients is required.

The main objective of this paper is to derive the explicit Jacobian matrix of HB algebraic system, which is used for sensitivity analysis. As applications to aeroelastic systems are targeted, three types of nonlinearities, cubic, freeplay and hysteresis, are considered.

The rest of this paper is organized as follows: a detailed description of the airfoil model used for simulation is given in Section 2. The general formulation of the developed method for determining the periodic solutions is presented in Section 3. Demonstrations of the proposed method are then conducted in Section 4 where numerical examples are given. Finally, concluding remarks are presented and discussed in Section 5.

2. The airfoil model

A two-degree-of-freedom pitch-and-plunge airfoil model is considered and the prototypical aeroelastic wing section is shown in Fig. 1. The plunge deflection is denoted by h, which is positive in the downward direction, and α is the pitch angle between the elastic axis and the positive nose up direction. The elastic axis is located at a distance $a_h b$ from the midchord, positive toward the trailing edge of the airfoil.