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Deformations of an elastic pipe submitted to gravity and internal fluid flow

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A R T I C L E I N F O

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A B S T R A C T

This paper describes the deformation of an elastic pipe submitted to gravity and to an internal fluid flow. The pipe is clamped horizontally at one end and free at the other end. As the fluid velocity increases, the shape changes from an elastic beam deflected by its own weight towards an horizontal position. The shape of the pipe is characterized experimentally and is compared with a theoretical model based on the Euler–Bernoulli approximation and the conservation of the fluid momentum. We study how the determination of the pipe deformation provides an estimation of the conveyed fluid flow. Finally, the vertical force produced by the conveyed fluid to lift off a mass is deduced.

1. Introduction

In 1895, Louis Lumière used a collapsible pipe to produce the joke of the first fictional movie *L'arroseur arrosé* (also known as *The Waterer Watered*) [Fig. 1]. In this movie, a boy steps the hose of a gardener watering his plants and produce the water cut. When the gardener looks the nozzle to inspect it, the boy unsteps the hose, causing the water to spray him.

The interaction between a compliant pipe and a fluid flow is also of first importance in heat exchangers of nuclear reactors (Païdoussis, 1980) and deep-ocean mining (Chung et al., 1981). This problem led to numerous contributions such as the study of the section modifications of a collapsible tube conveying a fluid flow by a numerical and theoretical approaches (Pedley and Luo, 1998; Rosar and Peskin, 2001; Tang et al., 2001). Above a critical fluid velocity, the free end of an elastic pipe spontaneously oscillates. This instability, first reported by Bourrières (1939), led to further investigations (Bajaj and Sethna, 1984; Doaré and de Langre, 2002; Castillo Flores and Cros, 2009; Kuronuma and Sano, 2003; Païdoussis and Issid, 1974; Païdoussis and Moon, 1988). The interaction of an elastic structure with an external fluid flow was inspected by Schouveiler et al. (2005) who consider the static equilibrium states and the stability of a flexible filament hanging in a steady and horizontal uniform air flow. However, no studies report the equilibrium state of an elastic structure submitted to an internal fluid flow.

In this paper, we study the static shape of an elastic pipe clamped horizontally at one end, the other end being free. The pipe conveys a water flow and is subjected to gravity. We observe that the flow straightens the pipe towards an horizontal position. First, measurements of the static shape of the pipe for various flow velocities are reported. Then, we describe theoretically this shape by considering the Bernoulli–Euler beam theory and the fluid inertia. This approach provides the

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Fig. 1. Picture of the first comedy of the history of the cinema produced by Louis Lumière in 1895: *L'arroseur arrosé* (also known as *The Waterer Watered*). This picture is provided by the Louis Lumière Institute in Lyon (http://www.institut-lumiere.org/musee/premiere-seance.html).

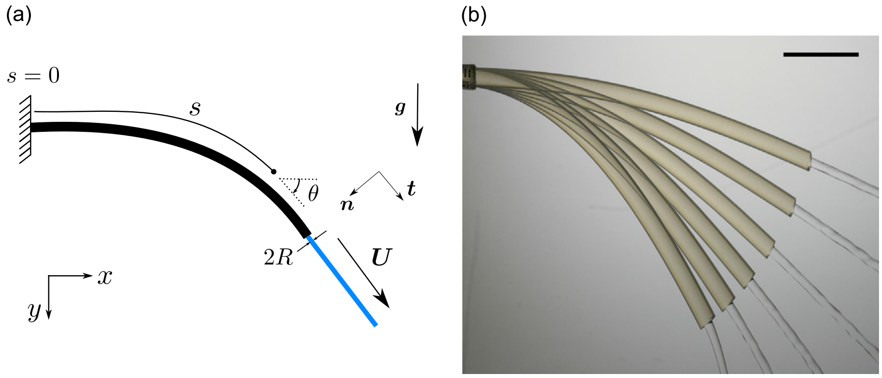

Fig. 2. (a) Sketch of the experimental setup. The curvilinear abscissa s is taken along the pipe direction, the angle θ with the horizontal direction x, g the gravitational acceleration, R the inner pipe radius and U the mean fluid velocity in the pipe. (b) Picture of an elastic pipe for various flow rates viewed from the side. The pipe length is $L=94$ mm, its inner radius $R=1.36$ mm, its total linear mass $\rho_p=21.7$ g/m, its flexural rigidity $EI=1.8\times10^{-5}$ N m^2 and the fluid velocities are $U=0.4, 1.3, 1.9, 2.5, 3.2, 3.8$ m/s respectively from bottom to up. Section 3 will introduce a characteristic length $l=\left(\frac{EI}{\rho_p g}\right)^{1/3}$ and a characteristic velocity $u=\frac{1}{L}\left(\frac{EI}{\rho_f}\right)^{1/2}$. Normalizing the pipe length and the fluid velocities by l and u provides $\overline{L}=L/l=2.3$ and $\overline{U}=U/u=0.6, 1.8, 2.7, 3.6, 4.6, 5.5$ respectively from bottom to up. The black line indicates 2 cm.

vertical force induced by the conveyed fluid. We discuss in what extend this force allows to lift off an external load located at the free end. Finally, we discuss the critical flow for which the free end of the pipe becomes unstable.

2. Experiments

An elastic tube of length $L=94$ mm, inner radius $R=1.36$ mm, linear mass $\rho_l=15.9$ g/m, bending momentum $I=3.3\times10^{-12}$ m^4 and Young modulus $E=5.4\times10^6$ N/m^2 is clamped horizontally from one end [Fig. 2]. The pipe conveys water of density $\rho=1.0$ g/cm^3 which corresponds to a linear mass $\rho_f=\rho\pi R^2=5.8$ g/m. Thus, the total linear mass of the pipe is $\rho_p=\rho_l+\rho_f=21.7$ g/m. The determination of the flexural rigidity $EI=1.8\times10^{-5}$ Nm^2 can be made by fitting the pipe deformations when no fluid is inside with the theoretical law expected for a cantilevered beam hanging under its own weight. Side-view pictures of the setup are taken for various flow rate Q ($2.3<Q<22$ cm^3/s). The fluid flow is provided by a rotary pump Ismatec® and measured with a balance and a timer. The different shapes of the tube for various flow rates are superimposed in Fig. 2(b). When the flow rate is low, the pipe is curved downwards and adopts the shape of an elastic beam of flexural rigidity EI deflected by its own weight added with the fluid mass. In this situation, the liquid falls nearly on the foot of the tube holder. As the flow rate increases, the pipe tends towards the horizontal and the liquid is ejected further than at low flow rate. Fig. 3 reports the measurements of the tube deformations extracted from these experiments.

3. Theoretical description

The system composed by the pipe and the conveyed fluid is submitted to three forces: the weight per unit length $\rho_p\boldsymbol{g}$, the centrifugal force per unit length due to the fluid flow $-\rho_f U^2\frac{d\theta}{ds}\boldsymbol{n}$ with U the mean velocity of the fluid inside the pipe (linked