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Crack repair using an elastic filler

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ABSTRACT

The effect of repairing a crack in an elastic body using an elastic filler is examined in terms of the stress intensity levels generated at the crack tip. The effect of the filler is to change the stress field singularity from order $1/r^{1/2}$ to $1/r^{(1-\lambda)}$ where r is the distance from the crack tip, and λ is the solution to a simple transcendental equation. The singularity power $(1-\lambda)$ varies from $\frac{1}{2}$ (the unfilled crack limit) to 1 (the fully repaired crack), depending primarily on the *scaled* shear modulus ratio γ_r defined by $G_2/G_1 = \gamma_r \varepsilon$, where $2\pi\varepsilon$ is the (small) crack angle, and the indices (1, 2) refer to base and filler material properties, respectively. The fully repaired limit is effectively reached for $\gamma_r \approx 10$, so that fillers with surprisingly small shear modulus ratios can be effectively used to repair cracks. This fits in with observations in the mining industry, where materials with G_2/G_1 of the order of 10^{-3} have been found to be effective for stabilizing the walls of tunnels. The results are also relevant for the repair of cracks in thin elastic sheets.

1. Introduction

Remarkably thin and relatively elastically weak spray-on liners (TSLs) have been found to be useful for stabilizing the rock walls of mining tunnels, see Carstens (2005), Stacey and Yu (2004), Kuijpers and Toper (2004), and Wojno and Kuijpers (1997). Such liners are sprayed onto the rock surface with a typical thickness of 4 mm and are very flexible (Young's modulus E typically 0.2 GPa) compared with rock (E typically 70 GPa) so that structural support, as in arching, cannot explain the phenomenon. Such arching effects decrease in proportion to the thickness of the liner and the ratio of Young's modulus, see Mason and Stacey (2008). A number of possible mechanisms have been suggested to explain the effectiveness of the TSLs, and some preliminary calculations and experiments have been carried out, see Stacey (2001) and Stacey and Yu (2004). Suggestions include 'basketing', promotion of block interlocking, suction support, and stress spreading, and it has been suggested that such mechanisms may act in concert. Attempts have also been made to numerically model the effect of coatings on fractured and stressed tunnels, see Wang and Tannant (2004), but the results are not revealing. To date no firm conclusions can be drawn.

One possible mechanism is that the sprayed on material fills cracks reaching the rock face and in this way reduces stress intensity levels at the crack tips, thus preventing or inhibiting crack propagation and consequent failure. Additionally the flexible liner may prevent the crack face separation required for rupture and facilitate the redistribution of stress away from the crack into the base material. We investigate these issues here. To do this we first obtain asymptotic results for the stress field in the neighbourhood of the filled crack tip. The stress field singularity is found to be sensitively dependent on the relative material properties and crack angle. In order to obtain explicit exact results for the effect of the external loading we then determine modified stress intensity factors for a filled Griffith crack.

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The main results are contained in Section 2. Some concluding remarks and further comments on the significance of the results obtained in the mining context complete the work in Section 3. The results obtained in this article are also relevant for the repair of cracks in thin sheets.

2. Wedge solutions

We first determine eigensolutions of the elastic equations in a region Ω consisting of two infinite adjoining wedges Ω_1 and Ω_2 with arbitrary solid wedge angles ω_1 and $\omega_2 = 2\pi - \omega_1$ made up of materials with different elastic properties, see Fig. 1. Later, we investigate the limit as $\omega_1 \to 2\pi$ and $\omega_2 \to 0$ to obtain the thin inclusion results. The large wedge angled material models the cracked rock and the inserted thin wedge models the filler. Although such adjoining wedge problems have been studied in the past, see Denisyuk (1992), the solution behaviour for the parameter range of particular interest for crack repair has not been addressed. For the planar situation of interest stresses can be described in terms of the Airy stress function Φ, where equilibrium and compatibility conditions require Φ to be biharmonic. The general solution for Φ can be obtained by appropriately combining and matching separated solutions Φ_i of the biharmonic equation in polar coordinates (r, θ) in the two adjoining wedge domains Ω_i, we have

$$\Phi_i = r^{\lambda+1}(X_i^1 c^+ + X_i^2 c^- + X_i^3 s^+ + X_i^4 s^-), \quad i = 1, 2, \tag{1}$$

where

$$c^\pm = \cos(\lambda \pm 1)\theta, \quad s^\pm = \sin(\lambda \pm 1)\theta \tag{2}$$

and the coefficients $X_i^1, \ldots$ are yet to be determined, see Mitchell (1899), or Timoshenko and Goodier (1970). The associated stress and displacement fields are given by

$$\begin{pmatrix} \sigma_r \\ \sigma_\theta \\ \tau_{r\theta} \end{pmatrix}_i = r^{\lambda-1} \begin{pmatrix} -c^+ & -(\lambda-3)c^- & s^+ & (\lambda-3)s^- \\ c^+ & (1+\lambda)c^- & -s^+ & -(1+\lambda)s^- \\ s^+ & (\lambda-1)s^- & c^+ & (\lambda-1)c^- \end{pmatrix} \begin{pmatrix} X_i^1 \\ X_i^2 \\ X_i^3 \\ X_i^4 \end{pmatrix} \tag{3}$$

and

$$\begin{pmatrix} u_r \\ u_\theta \end{pmatrix}_i = \frac{r^\lambda}{2\lambda G_i} \begin{pmatrix} -c^+ & -(\lambda-n_i)c^- & s^+ & (\lambda-n_i)s^- \\ s^+ & (\lambda+n_i)s^- & c^+ & (\lambda+n_i)c^- \end{pmatrix} \begin{pmatrix} X_i^1 \\ X_i^2 \\ X_i^3 \\ X_i^4 \end{pmatrix}. \tag{4}$$

Here $n_i = (3 - v_i)/(1 + v_i)$ for plane stress, and $n_i = (3 - 4v_i)$ for plane strain, where v_i are Poisson's ratios, and G_i are the shear moduli of the materials in the two domains ($E_i = 2G_i(1 + v_i)$). In the mining context the plane strain result is appropriate. The plane stress situation is appropriate for elastic sheets.

The eigenvalues for λ and the corresponding coefficients need to be determined so that the stresses $(\sigma_\theta, \tau_{r\theta})$ and the displacements (u_r, u_θ) are continuous across $\theta = \pm\omega_1/2$, the lines of contact between the two wedges Ω_1 and Ω_2, see Fig. 1; in context the filler is assumed to remain attached to the crack faces. This gives the following homogeneous equations for the coefficients $(X_i^1, X_i^2, X_i^3, X_i^4)$:

$$\begin{pmatrix} c_2^+ & (\lambda+1)c_2^- & -c_1^+ & (-\lambda-1)c_1^- \\ s_2^+ & (\lambda-1)s_2^- & -s_1^+ & (1-\lambda)s_1^- \\ c_2^+ & (\lambda-n_2)c_2^- & -g_r c_1^+ & -g_r(\lambda-n_1)c_1^- \\ s_2^+ & (\lambda+n_2)s_2^- & -g_r s_1^+ & -g_r(\lambda+n_1)s_1^- \end{pmatrix} \begin{pmatrix} X_2^1 \\ X_2^2 \\ X_1^1 \\ X_1^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{5}$$

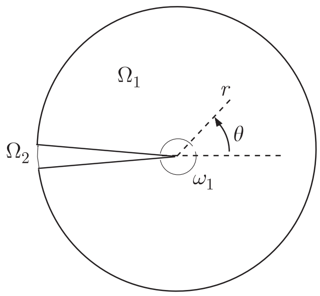

Fig. 1. Adjoining wedges Ω_1 (with solid angle ω_1), and Ω_2, with different elastic properties.