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An electromechanical liquid crystal model of vesicles

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ABSTRACT

An electromechanical liquid crystal model is developed for characterizing the equilibrium morphology of a lipid vesicle under coupled mechanical and electrical fields. A general equation that governs the vesicle shape is established, which incorporates the effects of elastic bending, osmotic pressure, surface tension, Maxwell pressure, as well as flexoelectric and dielectric properties of the lipid membrane. As an illustration of the model, the problem of an axisymmetric vesicle (e.g., a sphere or a cylinder) in a uniform electric field is considered in some detail, with results in agreement with relevant experimental results. The model provides an efficient tool for studying morphological evolution of dielectric vesicles under mechanical and electrical fields.

1. Introduction

Living cells can adapt to variations in the micro-environment by continuously altering their shapes and internal structures (Chen et al., 1997). Mechanical deformation of cells plays a significant role in various biological processes such as cell growth, differentiation, migration, and even apoptosis (Chen et al., 1997; Huang and Ingber, 1999; Boal, 2002; Schwartz and Ginsberg, 2002; Lim et al., 2006). For example, compression of chondrocytes was found to modulate proteoglycan synthesis (Buschmann et al., 1995; Bachrach et al., 1995), while stretching can alter both motility and orientation of cells on substrates (e.g., Liu, 1998). Red blood cells undergo very large deformation through blood vessels and narrow capillaries. On one hand, living cells are continuously subjected to mechanical stimulations in the environment. On the other hand, mechanical loads exerted at the tissue and organ levels are also transmitted to individual cells and influence their physiological functions (Guilak, 1995; Guilak and Mow, 2000). Mechanical properties of individual cells and their interactions with the extracellular matrix directly affect the structural integrity of tissue (Wakatsuki et al., 2000; Zahalak et al., 2000). The behaviors of cells in response to mechanical and electrical fields are of significance not only for gaining insights into the mechanical properties of cells, but also for understanding those of tissues and organs as a whole.

Recently, much attention has been focused on various phenomena and processes associated with cells and vesicles in electric fields, e.g., electroporation (Neumann, 1989; Schwan, 1989; Chang and Reese, 1990; Weaver, 1993; Weaver, 2003; Chen et al., 2006), electrofusion (Cevc and Richardsen, 1999), electrophoresis (Mehrishi and Bauer, 2002), electro-deformation and rotation (Neumann, 1989; Chassy, 1991; Lipowsky and Sackmann, 1995). Studies on the deformation behavior of cells under electric fields may provide efficient and quantitative techniques for cell control and manipulation (McCaig et al., 2005; Voldman, 2006), cell hybridization, cell migration, cell proliferation and differentiation, wound healing (McCaig et al., 2005), as well as delivery of foreign genes, proteins, antibodies, and drugs into cells (Weaver, 1993;

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Chen et al., 2006). Cells continuously adjust the properties (e.g., polarization property, dielectric property, and electrical conductivity) of cell membrane in response to surrounding micro-electrical fields to maintain their normal physiological functions (Zhao et al., 2004; Funk and Monsees, 2006). From this point of view, the morphological evolution of vesicles in an electric field should be of great interest to the understanding and control of electrophysiological properties of cells.

Continuum models of solids or fluids have often been adopted to study the responses of cells under mechanical forces (see, e.g., Bao and Suresh, 2003; Dao et al., 2003; Gao et al., 2005; Lim et al., 2006; Shi et al., 2006; and the references therein). Such models generally smear out the detailed microstructures of cell membrane with an average constitutive relation. Recently, discrete cytoskeleton models (Ingber, 1998; Boal, 2002) have also been suggested to study the motion and deformation of cells, in which cell components (e.g., actin filaments and microtubules) are abstracted as a network of cords, rods, and junctions with certain topological structures. These microstructure-based models manifest certain nonlinear mechanical behavior of cells (e.g., adhesion, large deformation, and complex dynamic properties of cells). However, many important properties of cell membranes such as instability, phase transformation, dielectric anisotropy, and flexoelectric effects have not yet been satisfactorily incorporated.

Using differential geometry and variational principle, Jenkins (1977a, b) derived the shape equation for a vesicle modeled as a fluid shell resisting bending. Experimental observations have demonstrated that the lipid bilayers of cell membranes are constructed based on the general principles of liquid crystals (Singer and Nicolson, 1972; Glenn and Wolken, 1979; Petrov, 1999; de Gennes and Prost, 1994). Therefore, the elastic theory of liquid crystal biomembranes has been successfully applied to study vesicle morphology, adhesion, and related problems (see, e.g., Seifert and Lipowsky, 1995; Seifert, 1997; Ou-Yang et al., 1999; Tu and Ou-Yang, 2004; Tu et al., 2006). This theory is based on Helfrich's curvature energy (Helfrich, 1973)

$$g_{\mathrm{H}} = \tfrac{1}{2}k(2H + c_0)^2 + k_{\mathrm{k}}K, \tag{1}$$

where k and k_{k} are elastic constants, H is the mean curvature, K the Gauss curvature, and c_0 the spontaneous curvature of membrane. Considering the bending, osmotic pressure and surface tension, Ou-Yang and Helfrich (1989) derived a more general shape equation of cell membranes:

$$\Delta p - 2\lambda H + k\nabla^2(2H) + k(2H + c_0)(2H^2 - c_0H - K) = 0, \tag{2}$$

which gave rise to both axisymmetric (e.g., sphere, column, ellipsoid, anchor ring, and dumbbell shapes) and nonaxisymmetric vesicle shapes (e.g., starfish, knizocyte, and sickle shapes; Ou-Yang et al., 1999). Similar methods have been used to study open lipid membranes and vesicles containing intramembrane domains with different elastic properties (Seifert and Lipowsky, 1995; Julicher and Lipowsky, 1996; Seifert, 1997; Ou-Yang et al., 1999; Capovilla and Guven, 2002; Tu and Ou-Yang, 2003; Umeda et al., 2005). Further studies have been directed toward the electric effects in vesicle deformation. Kummrow and Helfrich (1991) tested the deformation of spherical giant vesicles under an electric field and qualitatively analyzed the bending rigidity of vesicles. Hiroyuki et al. (1991a, b) studied the static and dynamic deformation of conductive vesicles in connection with Maxwell stresses on the inner and outer walls of the membrane. They concluded that a vesicle under axisymmetric loading generally possesses oblate or prolate ellipsoidal shapes, depending sensitively on the relative conductive coefficients inside and outside the vesicle. Based on a thin-shell theory and an energy model, Joshi et al. (2002a, b) performed a self-consistent theoretical analysis of cellular deformation in response to an applied quasistatic electric field. They found that although the ellipsoidal morphology can well describe the deformed cell shape at lower electric fields, the cell experiences large and thickness-dependent deformation at higher electric fields. Fan and Fedorov (2003) performed an analysis incorporating electrohydrodynamic and surface stress effects to study interactions between an AFM tip and a biomembrane in a dilute electrolyte solution.

As one of the important intrinsic properties of a liquid crystal system similar to piezoelectricity, the flexoelectric effect has a large influence on the deformation of cell membranes. Flexoelectricity refers to the coupling between the curvature and polarization of a membrane, or between the transmembrane voltage and membrane bending stress, which has been observed in experiments (Raphael et al., 2000) and related to mechanosensitivity and mechanotransduction of living systems (Petrov, 1999; Petrov, 2001). However, there has been surprisingly little investigation on the effect of flexoelectricity on the deformation of a cell membrane. Rey (2006) recently proposed a liquid crystal model accounting for the effects of pressure, tension, bending, torsion, and flexoelectric forces, however neglecting such effects as electrostatic pressure of the electrolytes and electric conductivity of cell membrane. Experiments have shown that vesicles can undergo shape transformation between sphere and prolate, prolate and oblate, or oblate and sphere, depending on the conductivity of medium and field frequency (Dimova et al., 2007). Moreover, when strong electric pulses are applied, some unusual deformation behaviors of vesicles changing among disc-, square- and tube-like shapes were observed (Riske and Dimova, 2006). These phenomena cannot be explained by a liquid crystal model that disregards the electrolyte and the electric conductivity of the cell membrane.

In response to recent advances in experimental observations, the present study is aimed at establishing a more general electromechanical liquid crystal model of cell membranes based on Eringen's micropolar theory (Eringen, 2001). The model accounts for contributions of elastic bending, osmotic pressure, surface tension, flexoelectric and dielectric effects under various types of mechanical and electrical fields. We will derive a set of general governing equations for the vesicle shape and make some detailed analysis of axisymmetric vesicles to illustrate the model.