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Brief Communication

Steady forces on a cylinder with oblique vortex shedding

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ABSTRACT

We present a curious situation of a fluid-flow wherein the body experiences non-fluctuating fluid-flow force despite being associated with an unsteady flow comprising of sustained vortex shedding. The flow past a circular cylinder at Re = 100 is investigated. It is shown that the spatio-temporal periodicity of the oblique vortex shedding results in constant-in-time force experienced by a cylinder placed in uniform flow. On the contrary, parallel vortex shedding leads to fluid force that fluctuates with time. It is found that, both, the parallel and oblique shedding are linearly unstable eigenmodes of the Re = 100 steady flow past a cylinder.

1. Introduction

The flow past a circular cylinder has received considerable attention in the last few decades (see, for example, the reviews by Williamson, 1996; Bearman, 1984; Berger and Wille, 1972; Zdravkovich, 1997). It involves rich physics and is associated with many interesting phenomena over a range of Reynolds number. The Reynolds number is defined as $Re = U_\infty D/\nu$. Here, U_∞ is the free-stream speed, D is the diameter of the cylinder and ν is the coefficient of kinematic viscosity.

Beyond $Re \sim 6.28$ the flow past a cylinder undergoes separation (Sen et al., 2009). This leads to the formation of two symmetric, counter-rotating vortices in the wake which increases in size with an increase in Re. The steady flow past a circular cylinder becomes unstable beyond $Re \sim 47$ via a Hopf bifurcation (Kumar and Mittal, 2006a, 2006b). After an initial linear growth, it achieves a state of limit cycle due to the non-linear processes (Verma and Mittal, 2011). The alternate shedding of vortices results in fluctuations in pressure and the von Karman vortex street (Williamson, 1996). In the classical configuration of the vortex street, for a nominally two-dimensional cylinder, the axes of the shed vortices are parallel to the axis of the cylinder. This is referred to as *parallel shedding*. In this situation, the body experiences unsteady force. Usually, the unsteady force in the transverse direction, with respect to the free-stream, is much larger compared to that in the in-line direction. This unsteady force may lead to noise/vibrations of the body. Verma and Mittal (2011) reported the another mode of wake instability for $Re > 110$, approximately. Beyond $Re \sim 180$, spanwise undulations appear in the wake marking the onset of three-dimensional instabilities (Williamson, 1996).

It has been observed in several laboratory experiments that the vortices can also be shed at an oblique angle to the axis of the cylinder (Berger and Wille, 1972; Tritton, 1971; Gerich and Eckelmann, 1982; Williamson, 1989). This is referred to as *oblique shedding*. The obliqueness of the vortices has been attributed to the conditions at the ends of the cylinder. The slant in the shed vortices may also be generated by the spanwise variation in the geometry of the cylinder as in the case of tapered or stepped cylinders (Visscher et al., 2011; Valles et al., 2002; Satish et al., 2013). It was demonstrated by Williamson

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(1989) that even for a cylinder of large aspect ratio (ratio of the spanwise length to diameter) the end-conditions influence the entire span of the cylinder. The oblique vortices, from both the ends of the cylinder, form a chevron pattern which is symmetric about the mid-span. It is possible to manipulate the end conditions to promote parallel shedding. This is achieved, for example, by inward angling of the leading edge of the end-plates (Williamson, 1989), or by placing control cylinders at the ends (Hammache and Gharib, 1991). The dependence of the oblique angle of the vortices on the conditions at the end-walls has also been investigated computationally (Mittal, 2001; Behara and Mittal, 2010). It has been shown that the oblique shedding angle varies linearly with the thickness of the boundary layer on the end wall. Allowing the velocity to slip on the end walls promotes parallel shedding. It was shown by Williamson (1989) that the vortex shedding frequency for the oblique and parallel shedding is related via the oblique angle of the vortices. Thus, a large number of vortex shedding states, each with a different oblique angle, are possible. This idea was utilized to explain the significant scatter in the measurement of Strouhal number from various experiments. The existence of different oblique shedding modes was also used to explain the discontinuity in the St−Re number curve at Re=64 (Williamson, 1989). Theoretical models, based on transverse stability theory (Albarède and Monkewitz, 1992; Triantafyllou, 1992; Leweke et al., 1997), have been further proposed to explain this phenomenon and the associated spanwise cells. Stability analysis (Noack and Eckelmann, 1994; Konig et al., 1993) has shown that three-dimensional unstable perturbations in the steady wake of cylinder are associated with discrete shedding modes. The focus, so far, in all the work related to oblique vortex shedding, has been toward understanding the onset of the slant in the wake, its modeling, its implications on the scatter in the data, and on techniques to induce parallel shedding. There has been no effort, that we are aware of, to understand the implications of the oblique vortex shedding on the fluid force acting on the body. It is also not clear from the earlier studies if the oblique and parallel shedding are related in any fundamental way. The primary objective of the present work is to investigate the effect of oblique shedding on the fluid force acting on the body and contrast it to the situation with parallel shedding. The other objective is to determine if any fundamental relationship between the two modes of vortex shedding exists, and utilize it to explain the difference in the time variation of fluid force in the two cases. We first show, via Direct Numerical Simulation (DNS) of the Re=100 flow past a cylinder, that the oblique vortex shedding is associated with spanwise periodicity. The fluid force, on a segment of the cylinder comprising of an integral number of spanwise wavelengths of the oblique waves, does not vary with time. The spanwise variation of the unsteady pressure field, during one cycle of vortex shedding, is studied to explain the distinction in the time-variation of force coefficients for oblique and parallel shedding. By carrying out a global linear stability analysis of the two-dimensional steady flow past a cylinder, we show that there are several unstable oblique modes with differing spanwise periodicity. We further show that the parallel mode is a special case of the oblique mode whose wavelength of periodicity along the span is infinite.

2. Results

A cylinder of aspect ratio AR=120, resides in a hexahedral domain. The x-axis is along the free stream flow while the z-axis is aligned with the axis of the cylinder. The cylinder occupies the entire span of the domain. Experiments carried out on high aspect ratio cylinders at Re=100, in the past, show that the vortices in the wake form a chevron pattern and posses symmetry along the centerline (Williamson, 1989). Taking advantage of the symmetry, only one half of the span is simulated and symmetry conditions are applied at the mid-span. It should be noted that the imposition of the symmetry boundary conditions, in the numerical set-up, does not interfere with the development of obliqueness in the vortices shed in the wake of the cylinder. The upstream and downstream boundaries are located at a distance of $50D$ and $100D$, respectively from the center of the cylinder, while the height of the domain is $100D$. Uniform flow is prescribed at the upstream face of the domain. At the outflow boundary the stress vector is set to zero. To promote oblique shedding, no-slip condition on the velocity is specified on one of the end walls ($z=0$) for $x/D \geq 5.0$. Symmetry conditions are prescribed on this boundary for $x/D < 5.0$. No-slip condition is also specified on the cylinder. Computations are carried out for the Re=100 flow with a mesh consisting of 14 million nodes and 13.7 million 8-noded hexahedral elements, approximately. A stabilized finite element formulation is utilized to solve the incompressible flow equations in primitive variables (Mittal, 2001). Equal-in-order interpolation functions are used for velocity and pressure. The simulation is carried out up to a non-dimensionalized time of 2500. $t=0$ corresponds to the onset of vortex shedding. Fully developed oblique shedding is attained at $t \approx 1200$ corresponding to 96 cycles of vortex shedding. Computations are also carried out for the case of parallel shedding. This is achieved by assigning symmetry conditions on both the end walls. The force on the cylinder, due to the fluid flow, is computed by integrating the stress, $\boldsymbol{\sigma}$, on its surface. The sectional force coefficients, per unit span, are defined as

$$C_l(z) = \frac{1}{\frac{1}{2}\rho U_\infty^2 D} \int_{\Gamma_{2D}} \boldsymbol{\sigma} \cdot \hat{n}_y \, d\Gamma, \quad C_d(z) = \frac{1}{\frac{1}{2}\rho U_\infty^2 D} \int_{\Gamma_{2D}} \boldsymbol{\sigma} \cdot \hat{n}_x \, d\Gamma. \tag{1}$$

Here, ρ is the density of fluid, $\hat{n}$ is the unit normal vector and Γ_{2D} is the circle lying on the cylinder at a spanwise location z. The lift and drag coefficients, for a finite span of the cylinder ($b = z_2 - z_1$), can be computed by integrating the sectional coefficients along span:

$$C_L = \frac{1}{b} \int_{z_1}^{z_2} C_l(z) \, dz, \quad C_D = \frac{1}{b} \int_{z_1}^{z_2} C_d(z) \, dz. \tag{2}$$