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Flow around four cylinders arranged in a square configuration

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ABSTRACT

This paper presents an experimental study of the flow around four circular cylinders arranged in a square configuration. The Reynolds number was fixed at $\mathrm{Re}=8000$, the pitch-to-diameter ratio between adjacent cylinders was varied from $P/D=2$ to 5 and the incidence angle was changed from $\alpha=0^\circ$ (in-line square configuration) to 45° (diamond configuration) at an interval of 7.5°. The flow field was measured using digital Particle Image Velocimetry (PIV) to examine the vortex shedding characteristics of the cylinders, together with direct measurement of fluid dynamic forces (lift and drag) on each cylinder using a piezoelectric load cell. Depending on the pitch ratio, the flow could be broadly classified as shielding regime ($P/D \le 2$), shear layer reattachment regime ($2.5 \le P/D \le 3.5$) and vortex impinging regime ($P/D \ge 4$). However, this classification is valid only in the case that the cylinder array is arranged nearly in-line with the free stream ($\alpha \approx 0^\circ$), because the flow is also sensitive to α. As α increases from 0° to 45°, each cylinder experiences a transition of vortex shedding pattern from a one-frequency mode to a two-frequency mode. The flow interference among the cylinders is complicated, which could be non-synchronous, quasi-periodic or synchronized with a definite phase relationship with other cylinders depending on the combined value of α and P/D. The change in vortex pattern is also reflected by some integral parameters of the flow such as force coefficients, power spectra and Strouhal numbers.

1. Introduction

The study of flow around circular cylindrical structures is of both fundamental and practical significance. A great deal of research work has been carried out in order to understand the classical problem of an isolated (or single) circular cylinder in cross flow, see the extensive reviews by Williamson (1996), Norberg (2003), Zdravkovich (2003) and Bearman (2011). On the other hand, in many cases, cylindrical structures are arranged in groups (or arrays) in applications for offshore structures, chimneys, power lines, heat exchanger tubes, etc. Due to mutual interference, the flow around multiple cylinders is usually a much more complicated, and thus is less well studied and understood than the case of an isolated cylinder.

The presence of neighboring cylinders significantly affects the wake flow patterns, force coefficients, vortex shedding frequencies, as well as vortex-induced vibration (VIV). Two cylinders arranged in various configurations (side-by-side, tandem, or staggered), being the simplest example of multiple-cylinder arrays, have attracted considerable attention over

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the last two decades and hence are now much better understood, as reviewed by Sumner (2010), in which about 150 technical papers were compiled.

The number of cylinders is obviously a critical parameter in determining the level of complexity in flow interference. As compared to the two-cylinder configuration, much less attention has been paid to the case of even larger number of cylinders, such as the three- or four-cylinder arrays, see for example the review by Khalifa et al. (2012) regarding the state-of-art research progresses on this topic. In particular, a four-cylinder array arranged in the square configuration is a fundamental element in offshore structures (e.g., semi-submersible platform), pipe bundles and tube banks. The geometry of the four-cylinder array is set by the spacing between the cylinders (which is typically expressed as the ratio between the cylinders' centre-to-centre pitch and the cylinder diameter, P/D, thereafter abbreviated as the pitch ratio) and the orientation angle of the cylinder array relative to the oncoming flow (α). When the cylinder's aspect ratio ($\mathrm{AR}=L/D$, where L is the length or span of the cylinder, and D is the cylinder diameter) is sufficiently large ($\mathrm{AR}\geq 8$ according to previous finding, e.g., Lam and Zou (2010)) that the flow can be treated as two-dimensional (2D), and is governed by three most important variables: (i) pitch ratio (P/D); (ii) incidence angle (α); and (iii) Reynolds number (Re), which is defined as $\mathrm{Re}=UD/\nu$, where U is the free-stream velocity and ν is the kinematic viscosity of fluid. To date, there are only a handful of reported experimental and numerical studies on the four-cylinder array, which are summarized in Table 1. The published studies, focusing on either the in-line square ($\alpha=0^\circ$) or the diamond ($\alpha=45^\circ$) configuration, revealed that the flow around four cylinders is far more complicated than the single- or two-cylinder counterparts. The interference among the cylinders in close proximity to one another may give rise to flow separation, reattachment, shear-layer instability and vortex impingement, as well as VIV (Zhao and Cheng, 2012). Clearly, the principle of superposition cannot be applied to this highly nonlinear situation.

The four-cylinder array in a square configuration was first investigated by Sayers (1988, 1990), who measured the force coefficients and vortex shedding frequencies for different pitch ratios and incidence angles at $\mathrm{Re}=3\times10^4$. Our present understanding of this flow configuration is largely attributed to the long-term research efforts by Lam and co-authors over the past 20 years (e.g., Lam and Lo, 1992; Lam and Fang, 1995; Lam et al., 2003a, 2003b; Lam et al., 2008; Lam and Zou, 2010). Most of the published experiments were conducted either in the laminar regime (Re = 100–200, mainly for flow visualization study) or in the subcritical regime ($\mathrm{Re}=10^3$–10^4, for measurements of pressure, velocity, drag and lift). On the other hand, it is noted that during the past several years, some numerical simulations have been carried out on the four-cylinder array at $\alpha=0^\circ$ or 45°. Except for the LES study by Lam and co-authors (Lam and Zou 2007, 2009; Zou et al., 2008) at $\mathrm{Re}=1.5\times10^4$ and the FEM study by Zhao and Cheng (2012) at $\mathrm{Re}=10^3$–2×10^4, most numerical studies were limited to relatively low Reynolds numbers ($\mathrm{Re}<300$), e.g., Farrant et al. (2000), Lam et al. (2008), Esfahani and Be Hagh (2010), Lam and Zou (2010), Tong et al. (2011), and Zou et al. (2011). Based on a flow visualization study conducted on the four-cylinder array at $\alpha=0^\circ$ (in-line square configuration), Lam and Lo (1992) classified the flow behavior into three distinct types of interference as a function of P/D: namely the shielding, reattachment and impinging regimes, which are shown in Fig. 1.

Because of the complexity of the multiple-cylinder wake, the vortex shedding frequency (f), typically expressed in non-dimensional form as the Strouhal number, $\mathrm{St}=fD/U$, may vary significantly across the wake. Lam et al. (2003b) showed that when P/D is less than 1.7, the flow around a four-cylinder array is biased (or deflected) behind the downstream cylinders, which is characterized by a narrow wake of higher frequency and a wide wake of lower frequency, as compared to that of a single cylinder. The different vortex shedding frequencies would be appropriately associated with individual shear layers

Table 1
Summary of published studies on four cylinders in a square configuration.

Researchers	Method	Technique[a]	Re	P/D	α (interval)
Sayers (1988, 1990)	Exp.	Pressure, HWA	3×10^4	1.1–5	0–180° (15°)
Lam and Lo (1992)	Exp.	Pressure, FV	2.1×10^3	1.28–5.96	0–45° (15°)
Lam and Fang (1995)	Exp.	Pressure	1.28×10^4	1.26–5.8	0–45° (15°)
Lam et al. (2003a)	Exp.	LIF, PIV	200	4	0–45° (5°)
Lam et al. (2003b)	Exp.	Force, LIF	200, 800 (LIF) 2.25–4.5×10^4 (Force)	1.69–3.83	0–45° (15°)
Lam and Zou (2007, 2009); Zou et al. (2008)	Exp.	LDA, PIV	1.1–2×10^4	1.5–5	0°
Lam et al. (2008)	Exp.	LIF	100, 200	1.6–5	0°
Present	Exp.	Force, PIV	8×10^3	2–5	0–45° (7.5°)
Farrant et al. (2000)	Sim.	BEM (2D)	200	3, 5	0°, 45°
Lam and Zou (2007, 2009); Zou et al. (2008)	Sim.	LES (3D)	1.5×10^4	1.5, 3.5	0°
Lam et al. (2008)	Sim.	FVM (2D, 3D)	100, 200	1.5, 3.5	0°
Esfahani and Be Hagh (2010)	Sim.	LBM (2D)	100	1.5–4.5	0°
Lam and Zou (2010)	Sim.	FVM (2D, 3D)	200	1.6–5	0°
Tong et al. (2011)	Sim.	FVM (2D, 3D)	270	2–6	0°
Zou et al. (2011)	Sim.	FVM (2D, 3D)	200	1.2–5	45°
Zhao and Cheng (2012)	Sim.	FEM (2D)	10^3–2×10^4	3	0–45° (15°)

[a] FV—flow visualization; HWA—hot-wire anemometry; LDA—laser Doppler anemometry; LIF—laser-induced fluorescence; PIV—Particle Image Velocimetry; BEM—Boundary Element Method; FEM—Finite Element Method; FVM—Finite Volume Method; LBM—Lattice Boltzmann Method; LES—Large eddy simulation.